Brayton Cycle with Intercooling, Reheating, and Regeneration

9-101C As the number of compression and expansion stages are increased and regeneration is employed, the ideal Brayton cycle will approach the Ericsson cycle.

9-102C (a) decrease, (b) decrease, and (c) decrease.

9-103C (a) increase, (b) decrease, and (c) decrease.

9-104C (a) increase, (b) decrease, (c) decrease, and (d) increase.

9-105C (a) increase, (b) decrease, (c) increase, and (d) decrease.

9-106C Because the steady-flow work is proportional to the specific volume of the gas. Intercooling decreases the average specific volume of the gas during compression, and thus the compressor work. Reheating increases the average specific volume of the gas, and thus the turbine work output.

9-107C (c) The Carnot (or Ericsson) cycle efficiency.
An ideal gas-turbine cycle with two stages of compression and two stages of expansion is considered. The back work ratio and the thermal efficiency of the cycle are to be determined for the cases of with and without a regenerator.

**Assumptions**

1. The air standard assumptions are applicable.
2. Air is an ideal gas with variable specific heats.
3. Kinetic and potential energy changes are negligible.

**Properties**

The properties of air are given in Table A-17.

**Analysis**

(a) The work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine since this is an ideal cycle. Then,

\[
T_1 = 300 \text{ K} \quad \Rightarrow \quad h_1 = 300.19 \text{ kJ/kg}
\]

\[
P_{r_2} = \frac{P_2}{P_1} = (3)(1.386) = 4.158 \quad \Rightarrow \quad h_2 = h_4 = 411.26 \text{ kJ/kg}
\]

\[
T_5 = 1200 \text{ K} \quad \Rightarrow \quad h_5 = h_7 = 1277.79 \text{ kJ/kg}
\]

\[
P_{r_6} = \frac{P_6}{P_5} = \left(\frac{1}{3}\right)(238) = 79.33 \quad \Rightarrow \quad h_6 = h_8 = 946.36 \text{ kJ/kg}
\]

\[
w_{C,in} = 2(h_2 - h_1) = 2(411.26 - 300.19) = 222.14 \text{ kJ/kg}
\]

\[
w_{T,out} = 2(h_5 - h_6) = 2(1277.79 - 946.36) = 662.86 \text{ kJ/kg}
\]

Thus,

\[
r_{bw} = \frac{w_{C,in}}{w_{T,out}} = \frac{222.14}{662.86} = 33.5\%
\]

\[
q_{in} = (h_5 - h_4) + (h_7 - h_6) = (1277.79 - 411.26) + (1277.79 - 946.36) = 1197.96 \text{ kJ/kg}
\]

\[
w_{net} = w_{T,out} - w_{C,in} = 662.86 - 222.14 = 440.72 \text{ kJ/kg}
\]

\[
\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{440.72}{1197.96} = 36.8\%
\]

(b) When a regenerator is used, \( r_{bw} \) remains the same. The thermal efficiency in this case becomes

\[
q_{regen} = \varepsilon(h_8 - h_4) = (0.75)(946.36 - 411.26) = 401.33 \text{ kJ/kg}
\]

\[
q_{in} = q_{in,old} - q_{regen} = 1197.96 - 401.33 = 796.63 \text{ kJ/kg}
\]

\[
\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{440.72}{796.63} = 55.3\%
\]
A gas-turbine cycle with two stages of compression and two stages of expansion is considered. The back work ratio and the thermal efficiency of the cycle are to be determined for the cases of with and without a regenerator.

Assumptions
1. The air standard assumptions are applicable.
2. Air is an ideal gas with variable specific heats.
3. Kinetic and potential energy changes are negligible.

Properties
The properties of air are given in Table A-17.

Analysis
(a) The work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine. Then,

\[ T_1 = 300 \text{ K} \rightarrow h_1 = 300.19 \text{ kJ/kg} \]
\[ P_{r1} = 1.386 \]

\[ P_{r2} = \frac{P_2}{P_1} \left( \frac{3}{1.386} \right) = 4.158 \rightarrow h_{2s} = h_{4s} = 411.26 \text{ kJ/kg} \]
\[ \eta_C = \frac{h_{2s} - h_1}{h_{2} - h_1} \rightarrow h_{2} = h_{4} = h_{1} + (h_{2s} - h_1) / \eta_C \]
\[ = 300.19 + (411.26 - 300.19) / (0.80) \]
\[ = 439.03 \text{ kJ/kg} \]

\[ T_5 = 1200 \text{ K} \rightarrow h_5 = h_2 = 1277.79 \text{ kJ/kg} \]
\[ P_{r5} = 238 \]

\[ P_s = \frac{P_6}{P_5} \left( \frac{1}{3} \right) (238) = 79.33 \rightarrow h_6 = h_8 = 946.36 \text{ kJ/kg} \]
\[ \eta_T = \frac{h_5 - h_6}{h_5 - h_{6s}} \rightarrow h_6 = h_8 = h_5 - \eta_T (h_5 - h_{6s}) \]
\[ = 1277.79 - (0.85) (1277.79 - 946.36) \]
\[ = 996.07 \text{ kJ/kg} \]

\[ w_{C,in} = 2(h_2 - h_1) = 2(439.03 - 300.19) = 277.68 \text{ kJ/kg} \]
\[ w_{T,out} = 2(h_5 - h_6) = 2(1277.79 - 996.07) = 563.44 \text{ kJ/kg} \]

Thus,
\[ r_{bw} = \frac{w_{C,in}}{w_{T,out}} = \frac{277.68 \text{ kJ/kg}}{563.44 \text{ kJ/kg}} = 49.3\% \]
\[ q_{in} = (h_5 - h_4) + (h_7 - h_6) = (1277.79 - 439.03) + (1277.79 - 996.07) = 1120.48 \text{ kJ/kg} \]
\[ w_{net} = w_{T,out} - w_{C,in} = 563.44 - 277.68 = 285.76 \text{ kJ/kg} \]
\[ \eta_{th} = \frac{w_{net}}{q_{in}} = \frac{285.76 \text{ kJ/kg}}{1120.48 \text{ kJ/kg}} = 25.5\% \]

(b) When a regenerator is used, \( r_{bw} \) remains the same. The thermal efficiency in this case becomes
\[ q_{regen} = \epsilon (h_8 - h_4) = (0.75)(996.07 - 439.03) = 417.78 \text{ kJ/kg} \]
\[ q_{in} = q_{in,old} - q_{regen} = 1120.48 - 417.78 = 702.70 \text{ kJ/kg} \]
\[ \eta_{th} = \frac{w_{net}}{q_{in}} = \frac{285.76 \text{ kJ/kg}}{702.70 \text{ kJ/kg}} = 40.7\% \]
A regenerative gas-turbine cycle with two stages of compression and two stages of expansion is considered. The minimum mass flow rate of air needed to develop a specified net power output is to be determined.

**Assumptions**
1. The air standard assumptions are applicable.
2. Air is an ideal gas with variable specific heats.
3. Kinetic and potential energy changes are negligible.

**Properties**
The properties of air are given in Table A-17.

**Analysis**
The mass flow rate will be a minimum when the cycle is ideal. That is, the turbine and the compressors are isentropic, the regenerator has an effectiveness of 100%, and the compression ratios across each compression or expansion stage are identical. In our case it is \( r_p = \sqrt{3} = 3 \). Then the work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine.

\[
\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{110,000 \text{ kJ/s}}{440.72 \text{ kJ/kg}} = 249.6 \text{ kg/s}
\]
Jet-Propulsion Cycles

9-112C The power developed from the thrust of the engine is called the propulsive power. It is equal to thrust times the aircraft velocity.

9-113C The ratio of the propulsive power developed and the rate of heat input is called the propulsive efficiency. It is determined by calculating these two quantities separately, and taking their ratio.

9-114C It reduces the exit velocity, and thus the thrust.

9-115E A turbojet engine operating on an ideal cycle is flying at an altitude of 20,000 ft. The pressure at the turbine exit, the velocity of the exhaust gases, and the propulsive efficiency are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The air standard assumptions are applicable. 3 Air is an ideal gas with constant specific heats at room temperature. 4 Kinetic and potential energies are negligible, except at the diffuser inlet and the nozzle exit. 5 The turbine work output is equal to the compressor work input.

Properties The properties of air at room temperature are $c_p = 0.24 \text{ Btu/lbm.R}$ and $k = 1.4$ (Table A-2Ea).

Analysis (a) For convenience, we assume the aircraft is stationary and the air is moving towards the aircraft at a velocity of $V_1 = 900 \text{ ft/s}$. Ideally, the air will leave the diffuser with a negligible velocity ($V_2 \approx 0$).

Diffuser:

\[ \dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system} \] (steady)

\[ \dot{E}_{in} = \dot{E}_{out} \\
\frac{h_1 + V_1^2}{2} = \frac{h_2 + V_2^2}{2} \\
0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \\
0 = c_p(T_2 - T_1) - V_1^2/2 \\
\]

\[ T_2 = T_1 + \frac{V_1^2}{2c_p} = 470 + \left(\frac{900 \text{ ft/s}}{2 \times 0.24 \text{ Btu/lbm.R}}\right)\left(\frac{1 \text{ Btu/lbm}}{25,037 \text{ ft}^2/\text{s}^2}\right) = 537.4 \text{ R} \]

\[ P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = (7 \text{ psia}) \left(\frac{537.3 \text{ R}}{470 \text{ R}}\right)^{1.4/0.4} = 11.19 \text{ psia} \]

Compressor:

\[ P_3 = P_1 = \left(r_p P_2\right) = (13)(11.19 \text{ psia}) = 145.5 \text{ psia} \]

\[ T_3 = T_2 \left(\frac{P_3}{P_2}\right)^{(k-1)/k} = (537.4 \text{ R})(13)^{0.4/1.4} = 1118.3 \text{ R} \]
Turbine:

\[ w_{\text{comp,in}} = w_{\text{turb,out}} \rightarrow h_3 - h_2 = h_4 - h_5 \rightarrow c_p(T_3 - T_2) = c_p(T_4 - T_5) \]

or,

\[ T_5 = T_4 - T_3 + T_2 = 2400 - 1118.3 + 537.4 = 1819.1 \text{ R} \]

\[ P_5 = P_4 \left( \frac{T_5}{T_4} \right)^{\frac{k}{k-1}} = (145.5 \text{ psia} \left( \frac{1819.1 \text{ R}}{2400 \text{ R}} \right)^{0.4/0.4} = 55.2 \text{ psia} \]

(b) Nozzle:

\[ T_6 = T_5 \left( \frac{P_6}{P_5} \right)^{\frac{k-1}{k}} = (1819.1 \text{ R} \left( \frac{7 \text{ psia}}{55.2 \text{ psia}} \right)^{0.4/1.4} = 1008.6 \text{ R} \]

\[ \dot{E}_\text{in} - \dot{E}_\text{out} = \Delta \dot{E}_\text{system} \phi_0 \text{ (steady)} \]

\[ \dot{E}_\text{in} = \dot{E}_\text{out} \]

\[ h_5 + \frac{V_5^2}{2} = h_6 + \frac{V_6^2}{2} \]

\[ 0 = h_6 - h_5 + \frac{V_6^2 - V_5^2}{2} \phi_0 \]

or,

\[ V_6 = \sqrt{\left( \frac{2(0.240 \text{ Btu/lbm} \cdot \text{R})(1819.1 - 1008.6)}{1 \text{ Btu/lbm}} \frac{25,037 \text{ ft}^2/s^2}{1 \text{ Btu/lbm}} \right) = 3121 \text{ ft/s} \}

(c) The propulsive efficiency is the ratio of the propulsive work to the heat input,

\[ w_p = (V_{\text{exit}} - V_{\text{inlet}}) \rho_{\text{aircraft}} \]

\[ = \left[ (3121 - 900) \text{ ft/s} \right] \left[ 900 \frac{\text{ ft/s}}{25,037 \text{ ft}^2/s^2} \right] = 79.8 \text{ Btu/lbm} \]

\[ q_{\text{in}} = h_4 - h_3 = c_p(T_4 - T_3) = (0.24 \text{ Btu/lbm} \cdot \text{R})(2400 - 1118.3)\text{R} = 307.6 \text{ Btu/lbm} \]

\[ \eta_p = \frac{w_p}{q_{\text{in}}} = \frac{79.8 \text{ Btu/lbm}}{307.6 \text{ Btu/lbm}} = 25.9\% \]
A turbojet engine operating on an ideal cycle is flying at an altitude of 20,000 ft. The pressure at the turbine exit, the velocity of the exhaust gases, and the propulsive efficiency are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. The air standard assumptions are applicable.
3. Air is an ideal gas with variable specific heats.
4. Kinetic and potential energies are negligible, except at the diffuser inlet and the nozzle exit.
5. The turbine work output is equal to the compressor work input.

**Properties**
The properties of air are given in Table A-17E.

**Analysis**
(a) For convenience, we assume the aircraft is stationary and the air is moving towards the aircraft at a velocity of \( V_1 = 900 \text{ ft/s} \). Ideally, the air will leave the diffuser with a negligible velocity \( (V_2 \approx 0) \).

**Diffuser:**
\[
h_1 = 112.20 \text{ Btu/lbm} \quad P_{r_1} = 0.8548
\]
\[
h_2 = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}
\]
\[
0 = h_2 - h_1 + \frac{V_2^2}{2} = \frac{1 \text{ Btu/lbm}}{25,037 \text{ ft}^2/\text{s}^2} = 128.48 \text{ Btu/lbm} \rightarrow P_{r_2} = 1.3698
\]
\[
P_2 = P_1 \left( \frac{P_{r_2}}{P_{r_1}} \right) = (7 \text{ psia}) \left( \frac{1.3698}{0.8548} \right) = 11.22 \text{ psia}
\]

**Compressor:**
\[
P_3 = P_4 = (r_p) \left( \frac{P_2}{P_1} \right) = (13)(11.22 \text{ psia}) = 145.8 \text{ psia}
\]
\[
P_{r_3} = \left( \frac{P_3}{P_2} \right) \frac{P_{r_2}}{P_{r_1}} = \left( \frac{145.8}{11.22} \right) (1.368) = 17.80 \rightarrow h_3 = 267.56 \text{ Btu/lbm}
\]

**Turbine:**
\[
h_4 = 617.22 \text{ Btu/lbm} \quad P_{r_4} = 367.6
\]
\[
W_{\text{comp,in}} = W_{\text{turb, out}}
\]
\[
h_3 - h_2 = h_4 - h_5
\]

or,
\[
h_5 = h_4 - h_3 + h_2 = 617.22 - 267.56 + 128.48 = 478.14 \text{ Btu/lbm} \rightarrow P_{r_5} = 142.7
\]
\[
P_5 = P_4 \left( \frac{P_{r_5}}{P_{r_4}} \right) = (145.8 \text{ psia}) \left( \frac{142.7}{367.6} \right) = 56.6 \text{ psia}
\]
(b) Nozzle:

\[
P_{r_s} = P_{r_s} \left( \frac{P_6}{P_5} \right) = (142.7) \left( \frac{7 \text{ psia}}{56.6 \text{ psia}} \right) = 17.66 \quad \rightarrow \quad h_6 = 266.93 \text{ Btu/lbm}
\]

\[\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{\text{system}} = \delta \quad \text{(steady)}\]

\[
\dot{E}_{in} = \dot{E}_{out}
\]

\[h_5 + V_5^2 / 2 = h_6 + V_6^2 / 2
\]

\[0 = h_6 - h_5 + \frac{V_6^2 - V_5^2}{2}\]

or,

\[V_6 = \sqrt{2(h_5 - h_6)} = \sqrt{(2)(478.14 - 266.93) \text{Btu/lbm} \left( \frac{25,037 \text{ ft}^2/s^2}{1 \text{ Btu/lbm}} \right)} = 3252 \text{ ft/s}
\]

(c) The propulsive efficiency is the ratio of the propulsive work to the heat input,

\[w_p = (V_{exit} - V_{inlet}) V_{aircraft}
\]

\[= [(3252 - 900) \text{ ft/s}] (900 \text{ ft/s}) \left( \frac{1 \text{ Btu/lbm}}{25,037 \text{ ft}^2/s^2} \right) = 84.55 \text{ Btu/lbm}
\]

\[q_{in} = h_4 - h_3 = 617.22 - 267.56 = 349.66 \text{ Btu/lbm}
\]

\[\eta_p = \frac{w_p}{q_{in}} = \frac{84.55 \text{ Btu/lbm}}{349.66 \text{ Btu/lbm}} = 24.2\%
\]
9-117 A turbojet aircraft flying at an altitude of 9150 m is operating on the ideal jet propulsion cycle. The velocity of exhaust gases, the propulsive power developed, and the rate of fuel consumption are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. The air standard assumptions are applicable.
3. Air is an ideal gas with constant specific heats at room temperature.
4. Kinetic and potential energies are negligible, except at the diffuser inlet and the nozzle exit.
5. The turbine work output is equal to the compressor work input.

**Properties**
The properties of air at room temperature are:

\[ c_p = 1.005 \text{ kJ/kg.K} \] and \[ k = 1.4 \text{ (Table A-2a)} \]

**Analysis**
(a) We assume the aircraft is stationary and the air is moving towards the aircraft at a velocity of \( V_1 = 320 \text{ m/s} \). Ideally, the air will leave the diffuser with a negligible velocity \( (V_2 \approx 0) \).

**Diffuser:**

\[
\begin{align*}
\dot{E}_\text{in} - \dot{E}_\text{out} &= \Delta \dot{E}_\text{system} \\
= 0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}
\end{align*}
\]

\[ 0 = c_p(T_2 - T_1) - \frac{V_1^2}{2} \]

\[ T_2 = T_1 + \frac{V_1^2}{2c_p} = 241 K + \left( 320 \text{ m/s} \right)^2 \left( 1 \text{ kJ/kg} \cdot \text{K} \right) \left( 1000 \text{ m}^2/\text{s}^2 \right) = 291.9 \text{ K} \]

\[ P_2 = P_1 \left( \frac{T_2}{T_1} \right)^{(k-1)/k} = (32 \text{ kPa}) \left( \frac{291.9 \text{ K}}{241 \text{ K}} \right)^{1.4/0.4} = 62.6 \text{ kPa} \]

**Compressor:**

\[ P_3 = P_4 = (r_p P_2) = (12)(62.6 \text{ kPa}) = 751.2 \text{ kPa} \]

\[ T_3 = T_2 \left( \frac{P_3}{P_2} \right)^{(k-1)/k} = (291.9 \text{ K})(12)^{0.4/1.4} = 593.7 \text{ K} \]

**Turbine:**

\[ w_{\text{comp,in}} = w_{\text{turb,out}} \rightarrow h_3 - h_2 = h_4 - h_5 \rightarrow c_p(T_3 - T_2) = c_p(T_4 - T_3) \]

\[ T_5 = T_4 - T_3 + T_2 = 1400 - 593.7 + 291.9 = 1098.2 \text{ K} \]

**Nozzle:**

\[ T_6 = T_4 \left( \frac{P_6}{P_4} \right)^{(k-1)/k} = (1400 \text{ K})(32 \text{ kPa}) \left( \frac{751.2 \text{ kPa}}{751.2 \text{ kPa}} \right)^{0.4/1.4} = 568.2 \text{ K} \]

\[ \dot{E}_\text{in} - \dot{E}_\text{out} = \Delta \dot{E}_\text{system} \\
= 0 = h_5 + \frac{V_5^2}{2} = h_6 + \frac{V_6^2}{2} \]

\[ 0 = h_6 - h_5 + \frac{V_6^2 - V_5^2}{2} \rightarrow 0 = c_p(T_6 - T_5) + \frac{V_5^2}{2} \]

**or,**

\[ V_6 = \sqrt{\left( 2 \right)(1.005 \text{ kJ/kg.K})(1098.2 - 568.2)(1000 \text{ m}^2/\text{s}^2)} = 1032 \text{ m/s} \]

(b) \[ \dot{W}_p = m(V_\text{exit} - V_\text{inlet}) = 60 \text{ kg/s} \left( 1032 - 320 \right) \text{m/s} = 13,670 \text{ kW} \]

(c) \[ \dot{Q}_\text{in} = m(h_4 - h_3) = mc_p(T_4 - T_3) = 60 \text{ kg/s} \left( 1.005 \text{ kJ/kg.K} \right)(1400 - 593.7) \text{K} = 48,620 \text{ kJ/s} \]

\[ \dot{m}_{\text{fuel}} = \frac{\dot{Q}_\text{in}}{HV} = \frac{48,620 \text{ kJ/s}}{42,700 \text{ kJ/kg}} = 1.14 \text{ kg/s} \]
9-118 A turbojet aircraft is flying at an altitude of 9150 m. The velocity of exhaust gases, the propulsive power developed, and the rate of fuel consumption are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The air standard assumptions are applicable. 3 Air is an ideal gas with constant specific heats at room temperature. 4 Kinetic and potential energies are negligible, except at the diffuser inlet and the nozzle exit.

**Properties** The properties of air at room temperature are $c_p = 1.005 \text{ kJ/kg.K}$ and $k = 1.4$ (Table A-2a).

**Analysis** (a) For convenience, we assume the aircraft is stationary and the air is moving towards the aircraft at a velocity of $V_1 = 320 \text{ m/s}$. Ideally, the air will leave the diffuser with a negligible velocity ($V_2 \cong 0$).

**Diffuser:**

\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta \dot{E}_{\text{system}} \quad \text{steady} \\
\dot{E}_{\text{in}} = \dot{E}_{\text{out}} \\
h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \\
0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \\
0 = c_p (T_2 - T_1) - \frac{V_1^2}{2}
\]

\[
T_2 = T_1 + \frac{V_1^2}{2c_p} = 241 \text{ K} + \frac{(320 \text{ m/s})^2}{(2)(1.005 \text{ kJ/kg.K})} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 291.9 \text{ K}
\]

\[
P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = (32 \text{ kPa}) \left(\frac{291.9 \text{ K}}{241 \text{ K}}\right)^{1.4/0.4} = 62.6 \text{ kPa}
\]

**Compressor:**

\[
P_3 = P_4 = (r_p) (P_2) = (12)(62.6 \text{ kPa}) = 751.2 \text{ kPa}
\]

\[
T_{3s} = T_2 \left(\frac{P_3}{P_2}\right)^{(k-1)/k} = (291.9 \text{ K})^{(12)^{0.4/1.4}} = 593.7 \text{ K}
\]

\[
\eta_C = \frac{h_3 - h_2}{h_3} = \frac{c_p (T_{3s} - T_2)}{h_3 - h_2} \\
T_3 = T_2 + (T_{3s} - T_2)/\eta_C = 291.9 + (593.7 - 291.9)(0.80) = 669.2 \text{ K}
\]

**Turbine:**

\[
w_{\text{comp,in}} = w_{\text{turb,out}} \quad h_3 - h_2 = h_4 - h_5 \quad c_p (T_3 - T_2) = c_p (T_4 - T_5)
\]

or,

\[
T_5 = T_4 - T_3 + T_2 = 1400 - 669.2 + 291.9 = 1022.7 \text{ K}
\]

\[
\eta_T = \frac{h_4 - h_5}{h_4} = \frac{c_p (T_4 - T_5)}{h_4} \\
T_{5s} = T_4 - (T_4 - T_5)/\eta_T = 1400 - (1400 - 1022.7)/0.85 = 956.1 \text{ K}
\]

\[
P_5 = P_4 \left(\frac{T_{5s}}{T_4}\right)^{k/(k-1)} = (751.2 \text{ kPa}) \left(\frac{956.1 \text{ K}}{1400 \text{ K}}\right)^{1.4/0.4} = 197.7 \text{ kPa}
\]
Nozzle:

\[ T_6 = T_5 \left( \frac{P_6}{P_5} \right)^{(k-1)/k} = (1022.7 \text{ K}) \left( \frac{32 \text{ kPa}}{197.7 \text{ kPa}} \right)^{0.4/1.4} = 607.8 \text{ K} \]

\[ \dot{E}_\text{in} - \dot{E}_\text{out} = \Delta E_\text{system} \]

\[ \dot{E}_\text{in} = \dot{E}_\text{out} \]

\[ h_5 + \frac{V_5^2}{2} = h_6 + \frac{V_6^2}{2} \]

\[ 0 = h_6 - h_5 + \frac{V_6^2 - V_5^2}{2} \]

\[ 0 = c_p (T_6 - T_5) + \frac{V_6^2}{2} \]

or,

\[ V_6 = \sqrt{\left(2 \right) \left(1.005 \text{ kJ/kg \cdot K} \right) \left(1022.7 - 607.8\right) \left( \frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)} = 913.2 \text{ m/s} \]

(b) \[ \dot{W}_p = \dot{m}(V_\text{exit} - V_\text{inlet})V_\text{aircraft} \]

\[ = (60 \text{ kg/s}) (913.2 - 320) \text{ m/s} (320 \text{ m/s}) \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \]

\[ = 11,390 \text{ kW} \]

(c) \[ \dot{Q}_\text{in} = \dot{m}(h_4 - h_3) = \dot{m}c_p(T_4 - T_3) = (60 \text{ kg/s}) \left(1.005 \text{ kJ/kg \cdot K} \right) (1400 - 669.2) \text{ K} = 44,067 \text{ kJ/s} \]

\[ \dot{m}_{\text{fuel}} = \frac{\dot{Q}_\text{in}}{H V} = \frac{44,067 \text{ kJ/s}}{42,700 \text{ kJ/kg}} = 1.03 \text{ kg/s} \]
9-119 A turbojet aircraft that has a pressure rate of 12 is stationary on the ground. The force that must be applied on the brakes to hold the plane stationary is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The air standard assumptions are applicable. 3 Air is an ideal gas with variable specific heats. 4 Kinetic and potential energies are negligible, except at the nozzle exit.

**Properties** The properties of air are given in Table A17.

**Analysis** (a) Using variable specific heats for air,

Compressor: \( T_1 = 300 \text{ K} \quad \rightarrow \quad h_1 = 300.19 \text{ kJ/kg} \)

\[
P_{r_2} = \frac{P_2}{P_1} = \left(12\right)\left(1.386\right) = 16.63 \quad \rightarrow \quad h_2 = 610.65 \text{ kJ/kg}
\]

\[
\dot{Q}_{\text{in}} = \dot{m}_{\text{fuel}} \times H = \left(0.2 \text{ kg/s}\right)\left(42700 \text{ kJ/kg}\right) = 8540 \text{ kJ/s}
\]

\[
\dot{q}_{\text{in}} = \frac{\dot{Q}_{\text{in}}}{\dot{m}} = \frac{8540 \text{ kJ/s}}{10 \text{ kg/s}} = 854 \text{ kJ/kg}
\]

\[
q_{\text{in}} = h_3 - h_2 \quad \rightarrow \quad h_3 = h_2 + q_{\text{in}} = 610.65 + 854 = 1464.65 \text{ kJ/kg}
\]

\[
\rightarrow \quad P_{r_3} = 396.27
\]

Turbine:

\[
w_{\text{comp,in}} = w_{\text{turb,out}} \quad \rightarrow \quad h_2 - h_1 = h_3 - h_4
\]

or,

\[
h_4 = h_3 - h_2 + h_1 = 1464.65 - 610.65 + 300.19 = 741.17 \text{ kJ/kg}
\]

Nozzle:

\[
P_{r_5} = P_s \left(\frac{P_3}{P_5}\right) = \left(396.27\right)\left(\frac{1}{12}\right) = 33.02 \quad \rightarrow \quad h_5 = 741.79 \text{ kJ/kg}
\]

\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta \dot{E}_{\text{system}} \quad \text{(steady)}
\]

\[
\dot{E}_{\text{in}} = \dot{E}_{\text{out}}
\]

\[
h_4 + \frac{V_4^2}{2} = h_5 + \frac{V_5^2}{2}
\]

\[
0 = h_5 - h_4 + \frac{V_5^2 - V_4^2}{2}
\]

or,

\[
V_5 = \sqrt{2\left(h_4 - h_5\right)} = \sqrt{\left(2\right)\left[\left(1154.19 - 741.17\right)\text{kJ/kg}\right] \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 908.9 \text{ m/s}
\]

Brake force = Thrust = \( \dot{m}\left(V_{\text{exit}} - V_{\text{inlet}}\right) = \left(10 \text{ kg/s}\right)\left(908.9 - 0\right)\text{m/s} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 9089 \text{ N} \)
Problem 9-119 is reconsidered. The effect of compressor inlet temperature on the force that must be applied to the brakes to hold the plane stationary is to be investigated.

**Analysis** Using EES, the problem is solved as follows:

\[
P_{\text{ratio}} = 12 \\
T_1 = 27 \, ^\circ\text{C} \\
P[1] = 95 \, \text{kPa} \\
V[1] = 0 \, \text{m/s} \\
HV_\text{fuel} = 42700 \, \text{kJ/kg} \\
m_{\text{dot}}_\text{fuel} = 0.2 \, \text{kg/s} \\
\eta_c = 1.0 \\
\eta_t = 1.0 \\
\eta_N = 1.0
\]

**Inlet conditions**

\[
h[1] = \text{ENTHALPY}(\text{Air}, T=T[1]) \\
s[1] = \text{ENTROPY}(\text{Air}, T=T[1], P=P[1]) \\
v[1] = \text{volume}(\text{Air}, T=T[1], P=P[1]) \\
m_{\text{dot}} = \frac{V_{\text{dot}}[1]}{V[1]}
\]

**Compressor analysis**

\[
P_{\text{ratio}} = \frac{P[2]}{P[1]} \]  
(T[2] is the isentropic value of T[1] at compressor exit)

\[
h_s[2] = \text{ENTHALPY}(\text{Air}, T=T_s[2]) \\
T_s[2] = \text{TEMPERATURE}(\text{Air}, s=s_s[2], P=P[2]) \\
\eta_c = \frac{h_s[2]-h[1]}{h[2]-h[1]} \]  
(Compressor adiabatic efficiency; \eta_c = \frac{W_{\text{dot}}_c_{\text{ideal}}}{W_{\text{dot}}_c_{\text{actual}}})

\[
m_{\text{dot}}*h[1] + W_{\text{dot}}_c = m_{\text{dot}}*h[2] \]  
(SSSF First Law for the actual compressor, assuming:
adiabatic, ke=pe=0)

**External heat exchanger analysis**

\[
(process 2-3 is SSSF constant pressure)

\[
h[3] = \text{ENTHALPY}(\text{Air}, T=T[3]) \\
Q_{\text{dot}}_\text{in} = m_{\text{dot}}_\text{fuel}*HV_\text{fuel} \\
m_{\text{dot}}*h[2] + Q_{\text{dot}}_\text{in} = m_{\text{dot}}*h[3] \]  
(SSSF First Law for the heat exchanger, assuming W=0, ke=pe=0)

**Turbine analysis**

\[
h_s[4] = \text{ENTROPY}(\text{Air}, T=T[4], P=P[3]) \\
(For the ideal case the entropies are constant across the turbine)

\[
P_{\text{ratio}} = \frac{P[3]}{P[4]} \]  

\[
h_s[4] = \text{ENTHALPY}(\text{Air}, h=h_s[4]) \\
\eta_t = \frac{W_{\text{dot}}_t}{W_{\text{ts}}_\text{dot}} \]  
(turbine adiabatic efficiency, W_{\text{ts}}_\text{dot} > W_{\text{dot}}_t)

\[
m_{\text{dot}}*h[3] = W_{\text{dot}}_t + m_{\text{dot}}*h[4] \]  
(SSSF First Law for the actual compressor, assuming:
adiabatic, ke=pe=0)

\[
T[4] = \text{TEMPERATURE}(\text{Air}, h=h[4]) \\
P[4] = \text{pressure}(\text{Air}, s=s_s[4], h=h_s[4])
\]

**Cycle analysis**

\[
W_{\text{dot}}_\text{net} = W_{\text{dot}}_t - W_{\text{dot}}_c \]  
(Definition of the net cycle work, kW)

\[
W_{\text{dot}}_\text{net} = 0 \, [\text{kW}]
\]
"Exit nozzle analysis:"
s₄ = entropy('air', T=T₄, P=P₄)
s₅ₛ = s₄  "For the ideal case the entropies are constant across the nozzle"

Tₛ₅ = TEMPERATURE(Air, s=s₅ₛ, P=P₅₅)  "Tₛ₅" is the isentropic value of T₅ at nozzle exit
hₛ₅ = ENTHALPY(Air, T=Tₛ₅)

Eta_N = (h₄ - h₅) / (h₄ - hₛ₅)
m_dot₄ * h₄ = m_dot₅ * (hₛ₅ + Velₛ₅^2/2*convert(m^2/s^2,kJ/kg))

"Brake Force to hold the aircraft:"

Thrust = m_dot₅*(Vel₅₅ - Vel₁)  "[N]"
BrakeForce = Thrust  "[N]"

"The following state points are determined only to produce a T-s plot"
T₂ = temperature('air', h=h₂)
s₂ = entropy('air', T=T₂, P=P₂)

<table>
<thead>
<tr>
<th>Brake Force [N]</th>
<th>m [kg/s]</th>
<th>T₃ [K]</th>
<th>T₁ [C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9971</td>
<td>11.86</td>
<td>1164</td>
<td>-20</td>
</tr>
<tr>
<td>9764</td>
<td>11.41</td>
<td>1206</td>
<td>-10</td>
</tr>
<tr>
<td>9568</td>
<td>10.99</td>
<td>1247</td>
<td>0</td>
</tr>
<tr>
<td>9383</td>
<td>10.6</td>
<td>1289</td>
<td>10</td>
</tr>
<tr>
<td>9207</td>
<td>10.24</td>
<td>1330</td>
<td>20</td>
</tr>
<tr>
<td>9040</td>
<td>9.9</td>
<td>1371</td>
<td>30</td>
</tr>
</tbody>
</table>
Air enters a turbojet engine. The thrust produced by this turbojet engine is to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. The air standard assumptions are applicable.
3. Air is an ideal gas with variable specific heats.
4. Kinetic and potential energies are negligible, except at the diffuser inlet and the nozzle exit.

**Properties**
The properties of air are given in Table A-17.

**Analysis**
We assume the aircraft is stationary and the air is moving towards the aircraft at a velocity of $V_1 = 300$ m/s. Taking the entire engine as our control volume and writing the steady-flow energy balance yield

$$ T_1 = 280 \text{ K} \quad \Rightarrow \quad h_1 = 280.13 \text{ kJ/kg} $$

$$ T_2 = 700 \text{ K} \quad \Rightarrow \quad h_2 = 713.27 \text{ kJ/kg} $$

$$ \dot{E}_\text{in} - \dot{E}_\text{out} = \Delta E_{\text{system}} \left(\varphi_0 \text{ (steady)}\right) $$

$$ \dot{E}_\text{in} = \dot{E}_\text{out} $$

$$ \dot{Q}_\text{in} + \dot{m}(h_1 + \frac{V_1^2}{2}) = \dot{m}(h_2 + \frac{V_2^2}{2}) $$

$$ \dot{Q}_\text{in} = \dot{m} \left( h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \right) $$

$$ 15,000 \text{ kJ/s} = (16 \text{ kg/s}) \left[ 713.27 - 280.13 + \frac{V_2^2 - (300 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) \right] $$

It gives $V_2 = 1048$ m/s

Thus,

$$ F_p = \dot{m}(V_2 - V_1) = (16 \text{ kg/s})(1048 - 300)\text{m/s} = 11,968 \text{ N} $$
Second-Law Analysis of Gas Power Cycles

9-122 The total exergy destruction associated with the Otto cycle described in Prob. 9-34 and the exergy at the end of the power stroke are to be determined.

**Analysis** From Prob. 9-34, \( q_{in} = 750 \), \( q_{out} = 357.62 \text{ kJ/kg} \), \( T_1 = 300 \text{ K} \), and \( T_4 = 774.5 \text{ K} \).

The total exergy destruction associated with this Otto cycle is determined from

\[
x_{\text{destroyed}} = T_0 \left( \frac{q_{out}}{T_L} - \frac{q_{in}}{T_H} \right) = (300 \text{ K}) \left( \frac{357.62 \text{ kJ/kg}}{300 \text{ K}} - \frac{750 \text{ kJ/kg}}{2000 \text{ K}} \right) = 245.12 \text{ kJ/kg}
\]

Noting that state 4 is identical to the state of the surroundings, the exergy at the end of the power stroke (state 4) is determined from

\[
\phi_4 = (u_4 - u_0) - T_0(s_4 - s_0) + P_0(v_4 - v_0)
\]

where

\[
\begin{align*}
&u_4 - u_0 = u_4 - u_1 = q_{out} = 357.62 \text{ kJ/kg} \\
&v_4 - v_0 = v_4 - v_1 = 0 \\
&s_4 - s_0 = s_4 - s_1 = s_4^* - s_1^* - R\ln \frac{P_4}{P_1} = s_4^* - s_1^* - R\ln \frac{T_4}{T_1} \\
&= 2.6823 - 1.70203 - (0.287 \text{ kJ/kg·K})\ln \frac{774.5 \text{ K}}{300 \text{ K}} = 0.7081 \text{ kJ/kg·K}
\end{align*}
\]

Thus,

\[
\phi_4 = (357.62 \text{ kJ/kg}) - (300 \text{ K})(0.7081 \text{ kJ/kg·K}) + 0 = 145.2 \text{ kJ/kg}
\]

9-123 The total exergy destruction associated with the Diesel cycle described in Prob. 9-47 and the exergy at the end of the compression stroke are to be determined.

**Analysis** From Prob. 9-47, \( q_{in} = 1019.7 \), \( q_{out} = 445.63 \text{ kJ/kg} \), \( T_1 = 300 \text{ K} \), \( \nu_1 = 0.906 \text{ m}^3/\text{kg} \), and \( \nu_2 = \nu_1 / r = 0.906 / 12 = 0.0566 \text{ m}^3/\text{kg} \).

The total exergy destruction associated with this Otto cycle is determined from

\[
x_{\text{destroyed}} = T_0 \left( \frac{q_{out}}{T_L} - \frac{q_{in}}{T_H} \right) = (300 \text{ K}) \left( \frac{445.63 \text{ kJ/kg}}{300 \text{ K}} - \frac{1019.7 \text{ kJ/kg}}{2000 \text{ K}} \right) = 292.7 \text{ kJ/kg}
\]

Noting that state 1 is identical to the state of the surroundings, the exergy at the end of the compression stroke (state 2) is determined from

\[
\phi_2 = (u_2 - u_0) - T_0(s_2 - s_0) + P_0(v_2 - v_0)
\]

\[
= (u_2 - u_1) - T_0(s_2 - s_1) + P_0(v_2 - v_1)
\]

\[
= (643.3 - 214.07) - 0 + (95 \text{ kPa})(0.0566 - 0.906) \text{ m}^3/\text{kg} \left( \frac{1 \text{ kJ}}{1 \text{ kPa·m}^3} \right)
\]

\[
= 348.6 \text{ kJ/kg}
\]
9-124E The exergy destruction associated with the heat rejection process of the Diesel cycle described in Prob. 9-49E and the exergy at the end of the expansion stroke are to be determined.

**Analysis** From Prob. 9-49E, \( q_{\text{out}} = 158.9 \text{ Btu/lbm} \), \( T_1 = 540 \text{ R} \), \( T_4 = 1420.6 \text{ R} \), and \( \nu_4 = \nu_1 \). At \( T_{\text{avg}} = (T_4 + T_1)/2 = (1420.6 + 540)/2 = 980.3 \text{ R} \), we have \( c_{u_{\text{avg}}} = 0.180 \text{ Btu/lbm} \cdot \text{R} \). The entropy change during process 4-1 is

\[
s_1 - s_4 = c_v \ln \frac{T_1}{T_4} + R \ln \frac{\nu_1}{\nu_4} = \left(0.180 \text{ Btu/lbm} \cdot \text{R}\right) \ln \frac{540 \text{ R}}{1420.6 \text{ R}} = -0.1741 \text{ Btu/lbm} \cdot \text{R}
\]

Thus,

\[
x_{\text{destroyed},41} = T_0 \left( s_1 - s_4 + \frac{q_{R,41}}{T_R} \right) = 540\text{R} \left( -0.1741\text{ Btu/lbm} \cdot \text{R} + \frac{158.9\text{ Btu/lbm}}{540\text{ R}} \right) = 64.9 \text{ Btu/lbm}
\]

Noting that state 4 is identical to the state of the surroundings, the exergy at the end of the power stroke (state 4) is determined from

\[
\phi_4 = (u_4 - u_0) - T_0 (s_4 - s_0) + P_0 (\nu_4 - \nu_0)
\]

where

\[
u_4 - \nu_0 = \nu_4 - \nu_1 = q_{\text{out}} = 158.9 \text{ Btu/lbm} \cdot \text{R}
\]

\[
\nu_4 - \nu_0 = \nu_4 - \nu_1 = 0
\]

\[
s_4 - s_0 = s_4 - s_1 = 0.1741 \text{ Btu/lbm} \cdot \text{R}
\]

Thus,

\[
\phi_4 = (158.9 \text{ Btu/lbm}) - (540\text{R} \cdot 0.1741 \text{ Btu/lbm} \cdot \text{R}) + 0 = 64.9 \text{ Btu/lbm}
\]

**Discussion** Note that the exergy at state 4 is identical to the exergy destruction for the process 4-1 since state 1 is identical to the dead state, and the entire exergy at state 4 is wasted during process 4-1.
9-125 The exergy destruction associated with each of the processes of the Brayton cycle described in Prob. 9-73 is to be determined.

**Analysis** From Prob. 9-73, \( q_{in} = 584.62 \text{ kJ/kg} \), \( q_{out} = 478.92 \text{ kJ/kg} \), and

\[
T_1 = 310 \text{ K} \quad \rightarrow \quad s_1^* = 1.73498 \text{kJ/kg} \cdot \text{K} \\
h_2 = 646.3 \text{kJ/kg} \quad \rightarrow \quad s_2^* = 2.47256 \text{kJ/kg} \cdot \text{K} \\
T_3 = 1160 \text{K} \quad \rightarrow \quad s_3^* = 3.13916 \text{kJ/kg} \cdot \text{K} \\
h_4 = 789.16 \text{kJ/kg} \quad \rightarrow \quad s_4^* = 2.67602 \text{kJ/kg} \cdot \text{K}
\]

Thus,

\[
\begin{align*}
\text{x}_{\text{destroyed,12}} &= T_0 \left( s_2^* - s_1^* \right) = T_0 \left( s_2^* - s_1^* - R \ln \frac{P_2}{P_1} \right) = 40.83 \text{ kJ/kg} \\
\text{x}_{\text{destroyed,23}} &= T_0 \left( s_3^* - s_2^* + \frac{q_{R,23}}{T_R} \right) = T_0 \left( s_3^* - s_2^* - R \ln \frac{P_3}{P_2} + \frac{q_{in}}{T_R} \right) = 87.35 \text{ kJ/kg} \\
\text{x}_{\text{destroyed,34}} &= T_0 \left( s_4^* - s_3^* \right) = T_0 \left( s_4^* - s_3^* - R \ln \frac{P_4}{P_3} \right) = 38.76 \text{ kJ/kg} \\
\text{x}_{\text{destroyed,41}} &= T_0 \left( s_1^* - s_4^* + \frac{q_{R,41}}{T_R} \right) = T_0 \left( s_1^* - s_4^* - R \ln \frac{P_1}{P_4} + \frac{q_{out}}{T_R} \right) = 206.0 \text{ kJ/kg}
\end{align*}
\]

9-126 The total exergy destruction associated with the Brayton cycle described in Prob. 9-93 and the exergy at the exhaust gases at the turbine exit are to be determined.

**Analysis** From Prob. 9-93, \( q_{in} = 601.94 \text{ kJ/kg} \), \( q_{out} = 279.68 \text{ kJ/kg} \), and \( h_6 = 579.87 \text{ kJ/kg} \).

The total exergy destruction associated with this Otto cycle is determined from

\[
\begin{align*}
\text{x}_{\text{destroyed}} &= T_0 \left( \frac{q_{out}}{T_L} - \frac{q_{in}}{T_H} \right) = (300 \text{ K}) \left( \frac{279.68 \text{ kJ/kg}}{300 \text{ K}} - \frac{601.94 \text{ kJ/kg}}{1800 \text{ K}} \right) = 179.4 \text{ kJ/kg}
\end{align*}
\]

Noting that \( h_0 = h_{\text{at 300 K}} = 300.19 \text{ kJ/kg} \), the stream exergy at the exit of the regenerator (state 6) is determined from

\[
\phi_6 = (h_6 - h_0) - T_0 (s_6 - s_0) + \frac{V_{6}^{2}}{2} + gz_{6}^{0}
\]

where \( s_6 - s_0 = s_6 - s_1 = s_0^* - s_1^* - R \ln \frac{P_5}{P_1} \) = 2.36275 - 1.70203 = 0.66072 \text{ kJ/kg} \cdot \text{K}

Thus, \( \phi_6 = 579.87 - 300.19 - (300 \text{ K})(0.66072 \text{ kJ/kg} \cdot \text{K}) = 81.5 \text{ kJ/kg} \)
Problem 9-126 is reconsidered. The effect of the cycle pressure on the total irreversibility for the cycle and the exergy of the exhaust gas leaving the regenerator is to be investigated.

Analysis

Using EES, the problem is solved as follows:

"Input data"
T_o = 300 [K]
T_L = 300 [K]
T_H = 1400 [K]
{Pratio = 10}
T[1] = 300 [K]
C_P=1.005 [kJ/kg-K]
P[1]= 100 [kPa]
P_o=P[1]
Eta_reg = 1.0
Eta_c =1.0"Compressor isentropic efficiency"
Eta_t =1.0"Turbine isentropic efficiency"

"Isentropic Compressor analysis"
"For the ideal case the entropies are constant across the compressor"
"T_s[2] is the isentropic value of T[2] at compressor exit"
T_s[2]=T[1]*(Pratio)^((k-1)/k)
Eta_c = w_compisen/w_comp
"compressor adiabatic efficiency, W_comp > W_compisen"

"Conservation of energy for the compressor for the isentropic case:

e_in - e_out = DELTAe=0 for steady-flow"
w_compisen = C_P*(T_s[2]-T[1])

"Actual compressor analysis:"
w_comp = C_P*(T[2]-T[1])

"External heat exchanger analysis"
"SSSF First Law for the heat exchanger, assuming W=0, ke=pe=0

e_in - e_out = DELTAe_cv =0 for steady flow"
q_in_noreg = C_P*(T[3]-T[2])
P[3]=P[2]"process 2-3 is SSSF constant pressure"

"Turbine analysis"
"For the ideal case the entropies are constant across the turbine"
T_s[4]=T[3]*/(1/Pratio)^((k-1)/k)
"T_s[4] is the isentropic value of T[4] at turbine exit"
Eta_t = w_turb /w_turbisen "turbine adiabatic efficiency, w_turbisen > w_turb"

"SSSF First Law for the isentropic turbine, assuming: adiabatic, ke=pe=0

e_in -e_out = DELTAe_cv = 0 for steady-flow"
w_turbisen=C_P*(T[3] - T_s[4])

"Actual Turbine analysis:"
w_turb= C_P*(T[3]-T[4])

"Cycle analysis"
w_net=w_turb-w_comp "[kJ/kg]"
Eta_th_noreg=w_net/q_in_noreg*Convert(, %) "(%)" 
"Cycle thermal efficiency"
Bwr=w_comp/w_turb "Back work ratio"
"With the regenerator the heat added in the external heat exchanger is"
\[ q_{\text{in\_withreg}} = C_P(T[3] - T[5]) \]
"The regenerator effectiveness gives \( h[5] \) and thus \( T[5] \) as:"
"Energy balance on regenerator gives \( h[6] \) and thus \( T[6] \) as:"
"Cycle thermal efficiency with regenerator"
\[ \text{Eta\_th\_withreg} = \frac{W_{\text{net}}}{q_{\text{in\_withreg}}} \times \text{Convert(\%, \%)} \]
"Irreversibility associated with the Brayton cycle is determined from:"
\[ q_{\text{out\_withreg}} = q_{\text{in\_withreg}} - W_{\text{net}} \]
\[ i_{\text{withreg}} = T_o \times (q_{\text{out\_withreg}} / T_L - q_{\text{in\_withreg}} / T_H) \]
\[ q_{\text{out\_noreg}} = q_{\text{in\_noreg}} - W_{\text{net}} \]
\[ i_{\text{noreg}} = T_o \times (q_{\text{out\_noreg}} / T_L - q_{\text{in\_noreg}} / T_H) \]
"Neglecting the ke and pe of the exhaust gases, the exergy of the exhaust gases at the exit of the regenerator is:"
\[ \Psi_{6} = (h[6] - h_o) - T_o(s[6] - s_o) \]
\[ \Psi_{\text{exit\_withreg}} = C_P(T[6] - T_o) - T_o(C_P \times \ln(T[6]/T_o) - R \times \ln(P[6]/P_o)) \]
\[ \Psi_{\text{exit\_noreg}} = C_P(T[4] - T_o) - T_o(C_P \times \ln(T[4]/T_o) - R \times \ln(P[4]/P_o)) \]

<table>
<thead>
<tr>
<th>( i_{\text{noreg}} )</th>
<th>( i_{\text{withreg}} )</th>
<th>( \text{Pratio} )</th>
<th>( \Psi_{\text{exit_noreg}} ) [kJ/kg]</th>
<th>( \Psi_{\text{exit_withreg}} ) [kJ/kg]</th>
<th>( \eta_{\text{th_noreg}} ) [%]</th>
<th>( \eta_{\text{th_withreg}} ) [%]</th>
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9-128 The exergy destruction associated with each of the processes of the Brayton cycle described in Prob. 9-98 and the exergy at the end of the exhaust gases at the exit of the regenerator are to be determined.

Properties

The gas constant of air is \( R = 0.287 \text{ kJ/kg·K} \) (Table A-1).

Analysis

From Prob. 9-98, \( q_{\text{in}} = 539.23 \text{ kJ/kg} \), \( q_{\text{out}} = 345.17 \text{ kJ/kg} \), and

\[
\begin{align*}
T_1 &= 300 \text{ K} \quad \Rightarrow s^*_1 = 1.70203 \text{ kJ/kg·K} \\
h_2 &= 586.04 \text{ kJ/kg} \quad \Rightarrow s^*_2 = 2.37348 \text{ kJ/kg·K} \\
T_3 &= 1200 \text{ K} \quad \Rightarrow s^*_3 = 3.17888 \text{ kJ/kg·K} \\
h_4 &= 797.88 \text{ kJ/kg} \quad \Rightarrow s^*_4 = 2.68737 \text{ kJ/kg·K} \\
h_5 &= 738.56 \text{ kJ/kg} \quad \Rightarrow s^*_5 = 2.60833 \text{ kJ/kg·K}
\end{align*}
\]

and, from an energy balance on the heat exchanger,

\[
h_5 - h_2 = h_4 - h_6 \quad \Rightarrow h_6 = 797.88 - 738.56 + 586.04 = 645.36 \text{ kJ/kg}
\]

\[
\Rightarrow s^*_6 = 2.47108 \text{ kJ/kg·K}
\]

Thus,

\[
x_{\text{destroyed,12}} = T_0 s_{\text{gen,12}} = T_0 (s_2 - s_1) = T_0 \left( s^*_2 - s^*_1 - R \ln \frac{P_2}{P_1} \right)
\]

\[
= (300 \text{ K}) \left( 2.37348 - 1.70203 - (0.287 \text{ kJ/kg·K}) \ln(8) \right) = 22.40 \text{ kJ/kg}
\]

\[
x_{\text{destroyed,34}} = T_0 s_{\text{gen,34}} = T_0 (s_4 - s_3) = T_0 \left( s^*_4 - s^*_3 - R \ln \frac{P_4}{P_3} \right)
\]

\[
= (300 \text{ K}) \left( 2.68737 - 3.17888 - (0.287 \text{ kJ/kg·K}) \ln(1/8) \right) = 31.59 \text{ kJ/kg}
\]

\[
x_{\text{destroyed,regen}} = T_0 s_{\text{gen,regen}} = T_0 \left( (s_5 - s_2) + (s_6 - s_4) \right) = T_0 \left( s^*_5 - s^*_2 + s^*_6 - s^*_4 \right)
\]

\[
= (300 \text{ K}) \left( 2.60833 - 2.37348 + 2.47108 - 2.68737 \right) = 5.57 \text{ kJ/kg}
\]

\[
x_{\text{destroyed,53}} = T_0 s_{\text{gen,53}} = T_0 \left( s_3 - s_5 - \frac{q_{R,53}}{T_R} \right) = T_0 \left( s^*_3 - s^*_5 - R \ln \frac{P_3}{P_5} - \frac{q_{\text{in}}}{T_H} \right)
\]

\[
= (300 \text{ K}) \left( 3.17888 - 2.60833 - \frac{539.23 \text{ kJ/kg}}{1260 \text{ K}} \right) = 42.78 \text{ kJ/kg}
\]

\[
x_{\text{destroyed,61}} = T_0 s_{\text{gen,61}} = T_0 \left( s_1 - s_6 + \frac{q_{R,61}}{T_R} \right) = T_0 \left( s^*_1 - s^*_6 - R \ln \frac{P_1}{P_6} + \frac{q_{\text{out}}}{T_L} \right)
\]

\[
= (300 \text{ K}) \left( 1.70203 - 2.47108 + \frac{345.17 \text{ kJ/kg}}{300 \text{ K}} \right) = 114.5 \text{ kJ/kg}
\]

Noting that \( h_0 = h_{300 \text{ K}} = 300.19 \text{ kJ/kg} \), the stream exergy at the exit of the regenerator (state 6) is determined from

\[
\phi_6 = (h_6 - h_0) - T_0 (s_6 - s_0) + \frac{V_0 \phi^0}{2} + g_6 \phi^0
\]

where \( s_6 - s_0 = s_6^* - s_1^* - R \ln \frac{P_6}{P_1} \phi^0 = 2.47108 - 1.70203 = 0.76905 \text{ kJ/kg·K} \)

Thus,

\[
\phi_6 = 645.36 - 300.19 - (300 \text{ K})(0.76905 \text{ kJ/kg·K}) = 114.5 \text{ kJ/kg}
\]
A gas-turbine plant uses diesel fuel and operates on simple Brayton cycle. The isentropic efficiency of the compressor, the net power output, the back work ratio, the thermal efficiency, and the second-law efficiency are to be determined.

**Assumptions**
1. The air-standard assumptions are applicable.
2. Kinetic and potential energy changes are negligible.
3. Air is an ideal gas with constant specific heats.

**Properties**
The properties of air at 500°C = 773 K are $c_p = 1.093 \text{ kJ/kg·K}$, $c_v = 0.806 \text{ kJ/kg·K}$, $R = 0.287 \text{ kJ/kg·K}$, and $k = 1.357$ (Table A-2b).

**Analysis**

(a) The isentropic efficiency of the compressor may be determined if we first calculate the exit temperature for the isentropic case

$$T_{2s} = T_1 \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = \left( 303 \text{ K} \right) \left( \frac{700 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.357-1)/1.357} = 505.6 \text{ K}$$

$$\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{505.6 - 303}{700 - 303} = 0.881$$

(b) The total mass flowing through the turbine and the rate of heat input are

$$\dot{m}_t = \dot{m}_a + \dot{m}_f = \dot{m}_a + \frac{\dot{m}_f}{AF} = 12.6 \text{ kg/s} + \frac{12.6 \text{ kg/s}}{60} = 12.6 \text{ kg/s} + 0.21 \text{ kg/s} = 12.81 \text{ kg/s}$$

$$\dot{Q}_{in} = \dot{m}_f q_{HV}\eta_c = (0.21 \text{ kg/s})(42,000 \text{ kJ/kg})(0.97) = 8555 \text{ kW}$$

The temperature at the exit of combustion chamber is

$$\dot{Q}_{in} = \dot{m} c_p (T_3 - T_2) \rightarrow 8555 \text{ kW/s} = (12.81 \text{ kg/s})(1.093 \text{ kJ/kg·K})(T_3 - 533 \text{ K}) \rightarrow T_3 = 1144 \text{ K}$$

The temperature at the turbine exit is determined using isentropic efficiency relation

$$T_{4s} = T_3 \left( \frac{P_4}{P_3} \right)^{(k-1)/k} = \left( 1144 \text{ K} \right) \left( \frac{700 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.357-1)/1.357} = 685.7 \text{ K}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} \rightarrow 0.85 \rightarrow \frac{(1144 - 74.4) \text{ K}}{(1144 - 685.7) \text{ K}} \rightarrow T_4 = 754.4 \text{ K}$$

The net power and the back work ratio are

$$\dot{W}_{C,in} = \dot{m}_a c_p (T_2 - T_1) = (12.6 \text{ kg/s})(1.093 \text{ kJ/kg·K})(533 - 303) \text{ K} = 3168 \text{ kW}$$

$$\dot{W}_{T,out} = \dot{m}_f c_p (T_5 - T_4) = (12.81 \text{ kg/s})(1.093 \text{ kJ/kg·K})(1144 - 754.4) \text{ K} = 5455 \text{ kW}$$

$$\dot{W}_{net} = \dot{W}_{T,out} - \dot{W}_{C,in} = 5455 - 3168 = 2287 \text{ kW}$$

$$\eta_{bw} = \frac{\dot{W}_{C,in}}{\dot{W}_{T,out}} = \frac{3168 \text{ kW}}{5455 \text{ kW}} = 0.581$$

(c) The thermal efficiency is

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{2287 \text{ kW}}{8555 \text{ kW}} = 0.267$$

The second-law efficiency of the cycle is defined as the ratio of actual thermal efficiency to the maximum possible thermal efficiency (Carnot efficiency). The maximum temperature for the cycle can be taken to be the turbine inlet temperature. That is,

$$\eta_{max} = 1 - \frac{T_1}{T_3} = 1 - \frac{303 \text{ K}}{1144 \text{ K}} = 0.735$$

and

$$\eta_{II} = \frac{\eta_{th}}{\eta_{max}} = \frac{0.267}{0.735} = 0.364$$
A modern compression ignition engine operates on the ideal dual cycle. The maximum temperature in the cycle, the net work output, the thermal efficiency, the mean effective pressure, the net power output, the second-law efficiency of the cycle, and the rate of exergy of the exhaust gases are to be determined.

**Assumptions**  
1. The air-standard assumptions are applicable.  
2. Kinetic and potential energy changes are negligible.  
3. Air is an ideal gas with constant specific heats.

**Properties**  
The properties of air at 850 K are:

- $c_p = 1.110 \text{ kJ/kg} \cdot \text{K}$,  
- $c_v = 0.823 \text{ kJ/kg} \cdot \text{K}$,  
- $R = 0.287 \text{ kJ/kg} \cdot \text{K}$,  
and  
- $k = 1.349$ (Table A-2b).

**Analysis**

(a) The clearance volume and the total volume of the engine at the beginning of compression process (state 1) are

\[
\frac{V_c + V_d}{V_c} = 14 = \frac{V_c + 0.0028 \text{ m}^3}{V_c} \rightarrow V_c = 0.002154 \text{ m}^3 = V_2 = V_x
\]

\[
V_1 = V_c + V_d = 0.002154 + 0.0028 = 0.003015 \text{ m}^3 = V_4
\]

Process 1-2: Isentropic compression

\[
T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{k-1} = (328 \text{ K})(14)^{1.349-1} = 823.9 \text{ K}
\]

\[
P_2 = P_1 \left( \frac{V_1}{V_2} \right)^k = (95 \text{ kPa})(14)^{1.349} = 3341 \text{ kPa}
\]

Process 2-x and x-3: Constant-volume and constant pressure heat addition processes:

\[
T_x = T_2 \frac{P_x}{P_2} = (823.9 \text{ K}) \frac{9000 \text{ kPa}}{3341 \text{ kPa}} = 2220 \text{ K}
\]

\[
q_{2-x} = c_v (T_x - T_2) = (0.823 \text{ kJ/kg} \cdot \text{K})(2220 - 823.9) \text{ K} = 1149 \text{ kJ/kg}
\]

\[
q_{2-x} = q_{x-3} = c_p (T_x - T_3) \rightarrow 1149 \text{ kJ/kg} = (0.823 \text{ kJ/kg} \cdot \text{K})(T_3 - 2220) \text{ K} \rightarrow T_3 = 3254 \text{ K}
\]

(b) $q_{in} = q_{2-x} + q_{x-3} = 1149 + 1149 = 2298 \text{ kJ/kg}$

\[
V_3 = V_x \frac{T_3}{T_x} = (0.0002154 \text{ m}^3) \frac{3254 \text{ K}}{2220 \text{ K}} = 0.0003158 \text{ m}^3
\]

Process 3-4: Isentropic expansion

\[
T_4 = T_3 \left( \frac{V_3}{V_4} \right)^{k-1} = (3254 \text{ K}) \left( \frac{0.0003158 \text{ m}^3}{0.0003015 \text{ m}^3} \right)^{1.349-1} = 1481 \text{ K}
\]

\[
P_4 = P_3 \left( \frac{V_3}{V_4} \right)^k = (9000 \text{ kPa}) \left( \frac{0.0003158 \text{ m}^3}{0.0003015 \text{ m}^3} \right)^{1.349} = 428.9 \text{ kPa}
\]

Process 4-1: Constant volume heat rejection.

\[
q_{out} = c_v (T_4 - T_1) = (0.823 \text{ kJ/kg} \cdot \text{K})(1481 - 328) \text{ K} = 948.7 \text{ kJ/kg}
\]

The net work output and the thermal efficiency are

\[
w_{net, out} = q_{in} - q_{out} = 2298 - 948.7 = 1349 \text{ kJ/kg}
\]

\[
\eta_{th} = \frac{w_{net, out}}{q_{in}} = \frac{1349 \text{ kJ/kg}}{2298 \text{ kJ/kg}} = 0.587
\]
(c) The mean effective pressure is determined to be

\[ m = \frac{P_1 V_1}{RT_1} = \frac{(95 \text{ kPa})(0.003015 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(328 \text{ K})} = 0.003043 \text{ kg} \]

\[ \text{MEP} = \frac{m \dot{w}_{\text{net, out}}}{V_1 - V_2} = \frac{(0.003043 \text{ kg})(1349 \text{ kJ/kg})}{(0.003015 - 0.0002154)\text{m}^3} \left( \frac{\text{kPa} \cdot \text{m}^3}{\text{kJ}} \right) = 1466 \text{ kPa} \]

(d) The power for engine speed of 3500 rpm is

\[ \dot{W}_{\text{net}} = \frac{m \dot{w}_{\text{net}}}{2} = \frac{(0.003043 \text{ kg})(1349 \text{ kJ/kg})}{(2 \text{ rev/cycle}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)} = 120 \text{ kW} \]

Note that there are two revolutions in one cycle in four-stroke engines.

(e) The second-law efficiency of the cycle is defined as the ratio of actual thermal efficiency to the maximum possible thermal efficiency (Carnot efficiency). We take the dead state temperature and pressure to be 25ºC and 100 kPa.

\[ \eta_{\text{max}} = 1 - \frac{T_0}{T_3} = 1 - \frac{(25 + 273) \text{ K}}{3254 \text{ K}} = 0.908 \]

and

\[ \eta_{\text{II}} = \frac{\eta_{\text{ab}}}{\eta_{\text{max}}} = \frac{0.587}{0.908} = 0.646 \]

The rate of exergy of the exhaust gases is determined as follows

\[ x_4 = u_4 - u_0 - T_0 (s_4 - s_0) = c_v(T_4 - T_0) - T_0 \left[ c_p \ln \frac{T_4}{T_0} - R \ln \frac{P_4}{P_0} \right] \]

\[ = (0.823)(1481 - 298) - (298) \left[ (1.110 \text{ kJ/kg.K}\ln \frac{1481}{298} - 0.287 \ln \frac{428.9}{100} \right] = 567.6 \text{ kJ/kg} \]

\[ \dot{X}_4 = m x_4 \frac{\dot{m}}{2} = (0.003043 \text{ kg})(567.6 \text{ kJ/kg}) \frac{3500 \text{ (rev/min)}}{(2 \text{ rev/cycle}) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)} = 50.4 \text{ kW} \]
A gas-turbine plant operates on the regenerative Brayton cycle. The isentropic efficiency of the compressor, the effectiveness of the regenerator, the air-fuel ratio in the combustion chamber, the net power output, the back work ratio, the thermal efficiency, the second law efficiency, the exergy efficiencies of the compressor, the turbine, and the regenerator, and the rate of the exergy of the combustion gases at the regenerator exit are to be determined.

**Assumptions**

1. The air-standard assumptions are applicable.
2. Kinetic and potential energy changes are negligible.
3. Air is an ideal gas with constant specific heats.

**Properties**

The properties of air at 500°C = 773 K are:
\[ c_p = 1.093 \text{ kJ/kg·K}, \quad c_v = 0.806 \text{ kJ/kg·K}, \quad R = 0.287 \text{ kJ/kg·K}, \quad k = 1.357 \] (Table A-2b).

**Analysis**

(a) For the compressor and the turbine:

\[
T_{2s} = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 303 \text{ K} \left( \frac{700 \text{ kPa}}{100 \text{ kPa}} \right)^{\frac{1.357-1}{1.357}} = 505.6 \text{ K}
\]

\[
\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{(505.6 - 303) \text{ K}}{(533 - 303) \text{ K}} = 0.881
\]

\[
T_{4s} = T_3 \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = 1144 \text{ K} \left( \frac{100 \text{ kPa}}{700 \text{ kPa}} \right)^{\frac{1.357-1}{1.357}} = 685.6 \text{ K}
\]

\[
\eta_F = \frac{T_3 - T_4}{T_4 - T_{4s}} = 0.85 = \frac{(1144 - 43) \text{ K}}{(1144 - 685.6) \text{ K}} \rightarrow T_4 = 754.4 \text{ K}
\]

(b) The effectiveness of the regenerator is:

\[
\varepsilon_{\text{regen}} = \frac{T_5 - T_2}{T_4 - T_{4s}} = \frac{(673 - 533) \text{ K}}{(754.4 - 533) \text{ K}} = 0.632
\]

(c) The fuel rate and air-fuel ratio are:

\[
\dot{Q}_{\text{in}} = \dot{m}_f \cdot q_{\text{HV}} \eta_c = (\dot{m}_f + \dot{m}_a) \cdot c_p \cdot (T_3 - T_5)
\]

\[
\dot{m}_f = 12.6 \text{ kg/s} \quad \Rightarrow \quad \dot{m}_f = 0.1613 \text{ kg/s}
\]

\[
\eta_f = \frac{12.6}{0.1613} = 78.14
\]

Also,

\[
\dot{m} = \dot{m}_a + \dot{m}_f = 12.6 + 0.1613 = 12.76 \text{ kg/s}
\]

\[
\dot{Q}_{\text{in}} = \dot{m}_f \cdot q_{\text{HV}} \eta_c = (0.1613 \text{ kg/s})(42,000 \text{ kJ/kg})(0.97) = 6570 \text{ kW}
\]

(d) The net power and the back work ratio are:

\[
\dot{W}_{\text{C,in}} = \dot{m}_a \cdot c_p \cdot (T_2 - T_1) = 12.6 \text{ kg/s}(1.093 \text{ kJ/kg·K})(533 - 303) \text{ K} = 3168 \text{ kW}
\]

\[
\dot{W}_{\text{T, out}} = \dot{m} c_p (T_3 - T_4) = 12.76 \text{ kg/s}(1.093 \text{ kJ/kg·K})(1144 - 754.4) \text{ K} = 5434 \text{ kW}
\]

\[
\dot{W}_{\text{net}} = \dot{W}_{\text{T, out}} - \dot{W}_{\text{C, in}} = 5434 - 3168 = 2267 \text{ kW}
\]

\[
r_{\text{bw}} = \frac{\dot{W}_{\text{C, in}}}{\dot{W}_{\text{T, out}}} = \frac{3168 \text{ kW}}{5434 \text{ kW}} = 0.583
\]

(e) The thermal efficiency is:

\[
\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{2267 \text{ kW}}{6570 \text{ kW}} = 0.345
\]

(f) The second-law efficiency of the cycle is defined as the ratio of actual thermal efficiency to the maximum possible thermal efficiency (Carnot efficiency). The maximum temperature for the cycle can be taken to be the turbine inlet temperature. That is,
\[ \eta_{\text{max}} = 1 - \frac{T_1}{T_3} = 1 - \frac{303 \text{ K}}{1144 \text{ K}} = 0.735 \]

and

\[ \eta_\Pi = \frac{\eta_{\text{max}}}{\eta_{\text{II}}} = \frac{0.345}{0.735} = 0.469 \]

(g) The exergy efficiency for the compressor is defined as the ratio of stream exergy difference between the inlet and exit of the compressor to the actual power input:

\[ \Delta \hat{X}_C = \dot{m}_a \left[ h_2 - h_1 - T_0 (s_2 - s_1) \right] = \dot{m}_a \left[ c_p \left( T_2 - T_1 \right) - T_0 \left[ c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right] \right] \]

\[ = (12.6) \left[ (1.093)(533 - 303) - (303)(1.093) \ln \left( \frac{533}{303} \right) - 0.287 \ln \left( \frac{700}{100} \right) \right] = 2943 \text{ kW} \]

\[ \eta_{\Pi,C} = \frac{\Delta \hat{X}_C}{W_{\text{C,in}}} = \frac{2943 \text{ kW}}{3168 \text{ kW}} = 0.929 \]

The exergy efficiency for the turbine is defined as the ratio of actual turbine power to the stream exergy difference between the inlet and exit of the turbine:

\[ \Delta \hat{X}_T = \dot{m} \left[ c_p \left( T_3 - T_4 \right) - T_0 \left[ c_p \ln \frac{T_3}{T_4} - R \ln \frac{P_3}{P_4} \right] \right] \]

\[ = (12.76) \left[ (1.093)(1144 - 754.4) - (303)(1.093) \ln \left( \frac{1144}{754.4} \right) - 0.287 \ln \left( \frac{700}{100} \right) \right] = 5834 \text{ kW} \]

\[ \eta_{\Pi,T} = \frac{W_{\text{T,in}}}{\Delta \hat{X}_T} = \frac{5434 \text{ kW}}{5834 \text{ kW}} = 0.932 \]

An energy balance on the regenerator gives

\[ \dot{m}_a c_p \left( T_5 - T_2 \right) = \dot{m}_c \left( T_4 - T_6 \right) \]

\[ (12.6)(1.093)(673 - 533) = (12.76)(1.093)(754.4 - T_6) \rightarrow T_6 = 616.2 \text{ K} \]

The exergy efficiency for the regenerator is defined as the ratio of the exergy increase of the cold fluid to the exergy decrease of the hot fluid:

\[ \Delta \hat{X}_{\text{regen,hot}} = \dot{m} \left[ c_p \left( T_4 - T_6 \right) - T_0 \left[ c_p \ln \frac{T_4}{T_6} - 0 \right] \right] \]

\[ = (12.76) \left[ (1.093)(754.4 - 616.2) - (303)(1.093) \ln \left( \frac{754.4}{616.2} \right) - 0 \right] = 1073 \text{ kW} \]

\[ \Delta \hat{X}_{\text{regen,cold}} = \dot{m} \left[ c_p \left( T_5 - T_2 \right) - T_0 \left[ c_p \ln \frac{T_5}{T_2} - 0 \right] \right] \]

\[ = (12.76) \left[ (1.093)(673 - 533) - (303)(1.093) \ln \left( \frac{673}{533} \right) - 0 \right] = 954.8 \text{ kW} \]

\[ \eta_{\Pi,\text{regen}} = \frac{\Delta \hat{X}_{\text{regen,cold}}}{\Delta \hat{X}_{\text{regen,hot}}} = \frac{954.8 \text{ kW}}{1073 \text{ kW}} = 0.890 \]

The exergy of the combustion gases at the regenerator exit:

\[ \hat{X}_6 = \dot{m} \left[ c_p \left( T_6 - T_0 \right) - T_0 \left[ c_p \ln \frac{T_6}{T_0} - 0 \right] \right] \]

\[ = (12.76) \left[ (1.093)(616.2 - 303) - (303)(1.093) \ln \left( \frac{616.2}{303} \right) - 0 \right] = 1351 \text{ kW} \]