Chapter 11
REFRIGERATION CYCLES

The Reversed Carnot Cycle

11-1C Because the compression process involves the compression of a liquid-vapor mixture which requires a compressor that will handle two phases, and the expansion process involves the expansion of high-moisture content refrigerant.

11-2 A steady-flow Carnot refrigeration cycle with refrigerant-134a as the working fluid is considered. The coefficient of performance, the amount of heat absorbed from the refrigerated space, and the net work input are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) Noting that $T_H = 30{}^\circ\text{C} = 303\text{ K}$ and $T_L = T_{sat @ 160\text{ kPa}} = -15.60{}^\circ\text{C} = 257.4\text{ K}$, the COP of this Carnot refrigerator is determined from

\[
\text{COP}_{R,C} = \frac{1}{T_H / T_L - 1} = \frac{1}{(303\text{ K})/(257.4\text{ K}) - 1} = 5.64
\]

(b) From the refrigerant tables (Table A-11),

\[
h_3 = h_{g@30{}^\circ\text{C}} = 266.66\text{ kJ/kg}
\]
\[
h_4 = h_{f@30{}^\circ\text{C}} = 93.58\text{ kJ/kg}
\]

Thus,

\[
q_H = h_3 - h_4 = 266.66 - 93.58 = 173.08\text{ kJ/kg}
\]

and

\[
\frac{q_H}{q_L} = \frac{T_H}{T_L} \rightarrow q_L = \frac{T_L}{T_H} q_H = \left(\frac{257.4\text{ K}}{303\text{ K}}\right)(173.08\text{ kJ/kg}) = 147.03\text{ kJ/kg}
\]

(c) The net work input is determined from

\[
w_{\text{net}} = q_H - q_L = 173.08 - 147.03 = 26.05\text{ kJ/kg}
\]
A steady-flow Carnot refrigeration cycle with refrigerant-134a as the working fluid is considered. The coefficient of performance, the quality at the beginning of the heat-absorption process, and the net work input are to be determined.

**Assumptions**

1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) Noting that $T_H = T_{sat@90\text{ psia}} = 72.78^\circ\text{F} = 532.8\text{ R}$ and $T_L = T_{sat@30\text{ psia}} = 15.37^\circ\text{F} = 475.4\text{ R}$.

$$\text{COP}_{R,C} = \frac{1}{T_H/T_L - 1} = \frac{1}{(532.8\text{ R})/(475.4\text{ R}) - 1} = 8.28$$

(b) Process 4-1 is isentropic, and thus

$$s_1 = s_4 = (s_f + x_4s_{fg})_{@90\text{ psia}} = 0.07481 + (0.05)(0.14525) = 0.08207\text{ Btu/lbm} \cdot \text{R}$$

$$x_1 = \frac{s_1 - s_f}{s_{fg} \@30\text{ psia}} = \frac{0.08207 - 0.03793}{0.18589} = 0.2374$$

(c) Remembering that on a $T$-$s$ diagram the area enclosed represents the net work, and $s_3 = s_g \@90\text{ psia} = 0.22006\text{ Btu/lbm} \cdot \text{R}$,

$$w_{\text{net,in}} = (T_H - T_L)(s_3 - s_4) = (72.78 - 15.37)(0.22006 - 0.08207)\text{ Btu/lbm} \cdot \text{R} = 7.92\text{ Btu/lbm}$$

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**Ideal and Actual Vapor-Compression Cycles**

**11-4C** Yes; the throttling process is an internally irreversible process.

**11-5C** To make the ideal vapor-compression refrigeration cycle more closely approximate the actual cycle.

**11-6C** No. Assuming the water is maintained at $10^\circ\text{C}$ in the evaporator, the evaporator pressure will be the saturation pressure corresponding to this pressure, which is 1.2 kPa. It is not practical to design refrigeration or air-conditioning devices that involve such extremely low pressures.

**11-7C** Allowing a temperature difference of $10^\circ\text{C}$ for effective heat transfer, the condensation temperature of the refrigerant should be $25^\circ\text{C}$. The saturation pressure corresponding to $25^\circ\text{C}$ is 0.67 MPa. Therefore, the recommended pressure would be 0.7 MPa.

**11-8C** The area enclosed by the cyclic curve on a $T$-$s$ diagram represents the net work input for the reversed Carnot cycle, but not so for the ideal vapor-compression refrigeration cycle. This is because the latter cycle involves an irreversible process for which the process path is not known.

**11-9C** The cycle that involves saturated liquid at $30^\circ\text{C}$ will have a higher COP because, judging from the $T$-$s$ diagram, it will require a smaller work input for the same refrigeration capacity.

**11-10C** The minimum temperature that the refrigerant can be cooled to before throttling is the temperature of the sink (the cooling medium) since heat is transferred from the refrigerant to the cooling medium.
11-11 A commercial refrigerator with refrigerant-134a as the working fluid is considered. The quality of the refrigerant at the evaporator inlet, the refrigeration load, the COP of the refrigerator, and the theoretical maximum refrigeration load for the same power input to the compressor are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

**Analysis**

(a) From refrigerant-134a tables (Tables A-11 through A-13)

\[
\begin{align*}
P_1 &= 60 \text{kPa} \\
T_1 &= -34^\circ \text{C} \\
P_2 &= 1200 \text{kPa} \\
T_2 &= 65^\circ \text{C} \\
P_3 &= 1200 \text{kPa} \\
T_3 &= 42^\circ \text{C} \\
h_1 &= 230.03 \text{kJ/kg} \\
h_2 &= 295.16 \text{kJ/kg} \\
h_3 &= 111.23 \text{kJ/kg} \\
h_4 &= h_3 = 111.23 \text{kJ/kg} \\
P_4 &= 60 \text{kPa} \\
h_4 &= 111.23 \text{kJ/kg}
\end{align*}
\]

Using saturated liquid enthalpy at the given temperature, for water we have (Table A-4)

\[
\begin{align*}
h_{w1} &= h_{f,18^\circ \text{C}} = 75.47 \text{kJ/kg} \\
h_{w2} &= h_{f,26^\circ \text{C}} = 108.94 \text{kJ/kg}
\end{align*}
\]

(b) The mass flow rate of the refrigerant may be determined from an energy balance on the compressor

\[
\dot{m}_R(h_2 - h_3) = \dot{m}_w(h_{w2} - h_{w1}) \\
\dot{m}_R(295.16 - 111.23) \text{kJ/kg} = (0.25 \text{ kg/s})(108.94 - 75.47) \text{kJ/kg}
\]

\[\dot{m}_R = 0.0455 \text{ kg/s}\]

The waste heat transferred from the refrigerant, the compressor power input, and the refrigeration load are

\[
\begin{align*}
\dot{Q}_H &= \dot{m}_R(h_2 - h_3) = (0.0455 \text{ kg/s})(295.16 - 111.23) \text{kJ/kg} = 8.367 \text{ kW} \\
\dot{W}_\text{in} &= \dot{m}_R(h_2 - h_1) - \dot{Q}_\text{in} = (0.0455 \text{ kg/s})(295.16 - 230.03) \text{kJ/kg} - 0.45 \text{ kW} = 2.513 \text{ kW} \\
\dot{Q}_L &= \dot{Q}_H - \dot{W}_\text{in} = 8.367 - 2.513 = 5.85 \text{ kW}
\end{align*}
\]

(c) The COP of the refrigerator is determined from its definition

\[\text{COP} = \frac{\dot{Q}_L}{\dot{W}_\text{in}} = \frac{5.85}{2.513} = 2.33\]

(d) The reversible COP of the refrigerator for the same temperature limits is

\[\text{COP}_{\text{max}} = \frac{1}{(T_H/T_L - 1) - 1} = \frac{1}{(18 + 273)/(30 + 273) - 1} = 5.063\]

Then, the maximum refrigeration load becomes

\[\dot{Q}_{L,\text{max}} = \text{COP}_{\text{max}} \dot{W}_\text{in} = (5.063)(2.513 \text{ kW}) = 12.72 \text{ kW}\]
An ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The rate of heat removal from the refrigerated space, the power input to the compressor, the rate of heat rejection to the environment, and the COP are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

Analysis (a) In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

\[ P_1 = 120 \text{ kPa} \quad h_1 = h_g \text{ at } 120 \text{ kPa} = 236.97 \text{ kJ/kg} \]

sat. vapor \[ s_1 = s_g \text{ at } 120 \text{ kPa} = 0.94779 \text{ kJ/kg} \cdot \text{K} \]

\[ P_2 = 0.7 \text{ MPa} \quad h_2 = 273.50 \text{ kJ/kg} \left( T_2 = 34.95^\circ\text{C} \right) \]

\[ s_2 = s_1 \]

\[ P_3 = 0.7 \text{ MPa} \quad h_3 = h_f \text{ at } 0.7 \text{ MPa} = 88.82 \text{ kJ/kg} \]

sat. liquid \[ h_4 \equiv h_3 = 88.82 \text{ kJ/kg} \text{ (throttling)} \]

Then the rate of heat removal from the refrigerated space and the power input to the compressor are determined from

\[ \dot{Q}_L = m(h_1 - h_4) = (0.05 \text{ kg/s})(236.97 - 88.82) \text{ kJ/kg} = 7.41 \text{ kW} \]

and

\[ W_{in} = m(h_2 - h_1) = (0.05 \text{ kg/s})(273.50 - 236.97) \text{ kJ/kg} = 1.83 \text{ kW} \]

(b) The rate of heat rejection to the environment is determined from

\[ \dot{Q}_H = \dot{Q}_L + W_{in} = 7.41 + 1.83 = 9.23 \text{ kW} \]

(c) The COP of the refrigerator is determined from its definition,

\[ \text{COP}_R = \frac{\dot{Q}_L}{W_{in}} = \frac{7.41 \text{ kW}}{1.83 \text{ kW}} = 4.06 \]
An ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The rate of heat removal from the refrigerated space, the power input to the compressor, the rate of heat rejection to the environment, and the COP are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**
(a) In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

\[
P_1 = 120 \text{ kPa} \quad h_1 = h_{g @120 \text{ kPa}} = 236.97 \text{ kJ/kg}
\]

sat. vapor \( s_1 = s_{g @120 \text{ kPa}} = 0.94779 \text{ kJ/kg} \cdot \text{K} \)

\[
P_2 = 0.9 \text{ MPa} \quad h_2 = 278.93 \text{ kJ/kg} \quad (T_2 = 44.45^\circ \text{C})
\]

\[
s_2 = s_1 \quad h_3 = h_{f @0.9 \text{ MPa}} = 101.61 \text{ kJ/kg}
\]

sat. liquid \( h_4 \approx h_3 = 101.61 \text{ kJ/kg} \) (throttling)

Then the rate of heat removal from the refrigerated space and the power input to the compressor are determined from

\[
\dot{Q}_L = m(h_1 - h_4) = (0.05 \text{ kg/s})(236.97 - 101.61) \text{ kJ/kg} = 6.77 \text{ kW}
\]

and

\[
\dot{W}_{in} = m(h_2 - h_1) = (0.05 \text{ kg/s})(278.93 - 236.97) \text{ kJ/kg} = 2.10 \text{ kW}
\]

(b) The rate of heat rejection to the environment is determined from

\[
\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 6.77 + 2.10 = 8.87 \text{ kW}
\]

(c) The COP of the refrigerator is determined from its definition,

\[
\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{6.77 \text{ kW}}{2.10 \text{ kW}} = 3.23
\]
An ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The throttling valve in the cycle is replaced by an isentropic turbine. The percentage increase in the COP and in the rate of heat removal from the refrigerated space due to this replacement are to be determined.

**Assumptions**

1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

If the throttling valve in the previous problem is replaced by an isentropic turbine, we would have

\[ x_{4s} = \left( \frac{s_3 - s_f}{s_{fg}} \right)_{@ 120 \text{ kPa}} = \frac{0.33230 - 0.09275}{0.85503} = 0.2802 \]

\[ h_{4s} = (h_f + x_{4s} h_{fg})_{@ 120 \text{ kPa}} = 22.49 + (0.2802)(214.48) = 82.58 \text{ kJ/kg} \]

Then, \( \dot{Q}_L = \dot{m}(h_1 - h_{4s}) = (0.05 \text{ kg/s})(236.97 - 82.58) \text{ kJ/kg} = 7.72 \text{ kW} \)

and \( \text{COP}_R = \frac{\dot{Q}_L}{W_{in}} = \frac{7.72 \text{ kW}}{1.83 \text{ kW}} = 4.23 \)

Then the percentage increase in \( \dot{Q} \) and COP becomes

\[ \frac{\Delta \dot{Q}_L}{\dot{Q}_L} = \frac{7.72 - 7.41}{7.41} = 4.2\% \]

\[ \frac{\Delta \text{COP}_R}{\text{COP}_R} = \frac{4.23 - 4.06}{4.06} = 4.2\% \]

**11-15** [Also solved by EES on enclosed CD] An ideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The quality of the refrigerant at the end of the throttling process, the COP, and the power input to the compressor are to be determined.

**Assumptions**

1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis (a)**

In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

\[ P_1 = 140 \text{ kPa} \]

\[ h_1 = h_g @ 140 \text{ kPa} = 239.16 \text{ kJ/kg} \]

sat. vapor \[ s_1 = s_g @ 140 \text{ kPa} = 0.94456 \text{ kJ/kg} \cdot \text{K} \]

\[ P_2 = 0.8 \text{ MPa} \]

\[ h_2 = 275.37 \text{ kJ/kg} \]

\[ s_2 = s_1 \]

\[ P_3 = 0.8 \text{ MPa} \]

sat. liquid \[ h_3 = h_f @ 0.8 \text{ MPa} = 95.47 \text{ kJ/kg} \]

\[ h_4 \leq h_3 = 95.47 \text{ kJ/kg} \text{ (throttling)} \]

The quality of the refrigerant at the end of the throttling process is

\[ x_4 = \left( \frac{h_4 - h_f}{h_{fg}} \right)_{@ 140 \text{ kPa}} = \frac{95.47 - 27.08}{212.08} = 0.322 \]

\[ \text{COP}_R = \frac{\dot{Q}_L}{W_{in}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{239.16 - 95.47}{275.37 - 239.16} = 3.97 \]

\[ W_{in} = \frac{\dot{Q}_L}{\text{COP}_R} = \frac{(300 / 60) \text{ kW}}{3.97} = 1.26 \text{ kW} \]
11-16 EES Problem 11-15 is reconsidered. The effect of evaporator pressure on the COP and the power input is to be investigated.

**Analysis** The problem is solved using EES, and the solution is given below.

"Input Data"

<table>
<thead>
<tr>
<th>Fluid</th>
<th>P[1] = 140 [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P[2]  = 800 [kPa]</td>
<td></td>
</tr>
<tr>
<td>Fluid$='R134a'</td>
<td></td>
</tr>
<tr>
<td>Eta_c=1.0 &quot;Compressor isentropic efficiency&quot;</td>
<td></td>
</tr>
<tr>
<td>Q_dot_in=300/60 &quot;[kJ/s]&quot;</td>
<td></td>
</tr>
</tbody>
</table>

"Compressor"

\[ x[1]=1 \ "assume inlet to be saturated vapor" \]
\[ h[1]=\text{enthalpy}($\text{Fluid$, P=P[1], x=x[1]}$) \]
\[ T[1]=\text{temperature}($\text{Fluid$, h=h[1], P=P[1]}$) "properties for state 1" \]
\[ s[1]=\text{entropy}($\text{Fluid$, T=T[1], x=x[1]}$) \]
\[ h[2]=\text{enthalpy}($\text{Fluid$, P=P[2], s=s[1]}$) "Identifies state 2 as isentropic" \]
\[ h[1]+Wc_\text{cs}=h[2] "energy balance on isentropic compressor" \]
\[ Wc=Wc_\text{cs}/\text{Eta_c} "definition of compressor isentropic efficiency" \]
\[ s[2]=\text{entropy}($\text{Fluid$, h=h[2], P=P[2]}$) "properties for state 2" \]
\[ T[2]=\text{temperature}($\text{Fluid$, h=h[2], P=P[2]}$) \]
\[ W_\text{dot}_c=m_\text{dot}*Wc \]

"Condenser"

\[ T[3]=\text{temperature}($\text{Fluid$, h=h[3], P=P[3]}$) "properties for state 3" \]
\[ h[3]=\text{enthalpy}($\text{Fluid$, P=P[3], x=0}$) "properties for state 3" \]
\[ s[3]=\text{entropy}($\text{Fluid$, T=T[3], x=0}$) \]
\[ h[2]=q_\text{out}+h[3] "energy balance on condenser" \]
\[ Q_\text{dot}_\text{out}=m_\text{dot}*q_\text{out} \]

"Valve"

\[ x[4]=\text{quality}($\text{Fluid$, h=h[4], P=P[4]}$) "properties for state 4" \]
\[ s[4]=\text{entropy}($\text{Fluid$, h=h[4], P=P[4]}$) \]
\[ T[4]=\text{temperature}($\text{Fluid$, h=h[4], P=P[4]}$) \]

"Evaporator"

\[ q_\text{in}+h[4]=h[1] "energy balance on evaporator" \]
\[ Q_\text{dot}_\text{in}=m_\text{dot}*q_\text{in} \]
\[ \text{COP}\_\text{plot}=Q_\text{dot}_\text{in}/W_\text{dot}_c "definition of COP" \]
\[ W_\text{dot}_\text{in}=W_\text{dot}_c \]

<table>
<thead>
<tr>
<th>P_1 [kPa]</th>
<th>COP_p \text{plot}</th>
<th>W_\text{b} [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>3.216</td>
<td>1.554</td>
</tr>
<tr>
<td>175</td>
<td>4.656</td>
<td>1.074</td>
</tr>
<tr>
<td>250</td>
<td>6.315</td>
<td>0.7918</td>
</tr>
<tr>
<td>325</td>
<td>8.388</td>
<td>0.5961</td>
</tr>
<tr>
<td>400</td>
<td>11.15</td>
<td>0.4483</td>
</tr>
</tbody>
</table>

**Note:** The COP and power data are given for different pressures at the evaporator. The COP increases with an increase in pressure, while the power input decreases. This indicates an improvement in the system efficiency due to the pressure effect.
R134a

T-s diagram for $\eta = 1.0$

-50  -25   25    50    75   100   125
 0,0  0,2  0,4  0,6  0,8  1,0  1,2

s [kJ/kg-K]

800 kPa
140 kPa

R134a

P-h diagram for $\eta = 1.0$

-50  -25   25    50    75   100   125
 0  50  100  150  200  250  300

h [kJ/kg]

$P = 31.33 \text{ C}$
$P = -18.8 \text{ C}$

R134a

T-s diagram for $\eta = 0.6$

-50  -25   25    50    75   100   125
 0,0  0,2  0,4  0,6  0,8  1,0  1,2

s [kJ/kg-K]

800 kPa
140 kPa

800 kPa
140 kPa
A nonideal vapor-compression refrigeration cycle with refrigerant-134a as the working fluid is considered. The quality of the refrigerant at the end of the throttling process, the COP, the power input to the compressor, and the irreversibility rate associated with the compression process are to be determined.

Assumptions

1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

Analysis

(a) The refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

\[
P_1 = 140 \text{ kPa} \quad h_1 = h_g @ 140 \text{ kPa} = 239.16 \text{ kJ/kg}
\]

sat. vapor \[ s_1 = s_g @ 140 \text{ kPa} = 0.94456 \text{ kJ/kg} \cdot \text{K} \]

\[
P_2 = 0.8 \text{ MPa} \quad h_{2s} = 275.37 \text{ kJ/kg}
\]

\[
s_2 = s_1 \quad \eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}
\]

\[
h_2 = h_1 + \left( h_{2s} - h_1 \right) / \eta_C
\]

\[
= 239.16 + (275.37 - 239.16) / (0.85)
\]

\[
= 281.76 \text{ kJ/kg}
\]

\[
P_3 = 0.8 \text{ MPa} \quad h_3 = h_f @ 0.8 \text{ MPa} = 95.47 \text{ kJ/kg}
\]

sat. liquid \[ h_4 = h_3 = 95.47 \text{ kJ/kg} \text{ (throttling)} \]

The quality of the refrigerant at the end of the throttling process is

\[
x_4 = \frac{h_4 - h_f}{h_g - h_f} @ 140 \text{ kPa} = \frac{95.47 - 27.08}{212.08} = 0.322
\]

(b) The COP of the refrigerator is determined from its definition,

\[
\text{COP}_R = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_2 - h_1}
\]

\[
= \frac{239.16 - 95.47}{281.76 - 239.16} = 3.37
\]

(c) The power input to the compressor is determined from

\[
W_{in} = \frac{\dot{Q}_L}{\text{COP}_R} = \frac{5 \text{ kW}}{3.37} = 1.48 \text{ kW}
\]

The exergy destruction associated with the compression process is determined from

\[
\dot{X}_{\text{destroyed}} = T_0 \dot{S}_{\text{gen}} = T_0 \dot{m} \left( s_2 - s_1 + \frac{q_{\text{sur}}}{T_0} \right) = T_0 \dot{m} (s_2 - s_1)
\]

where

\[
\dot{m} = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{5 \text{ kJ/s}}{(239.16 - 95.47) \text{ kJ/kg}} = 0.0348 \text{ kg/s}
\]

\[
P_2 = 0.8 \text{ MPa}
\]

\[
h_2 = 281.76 \text{ kJ/kg}
\]

\[
s_2 = 0.96483 \text{ kJ/kg} \cdot \text{K}
\]

Thus,

\[
\dot{X}_{\text{destroyed}} = (298 \text{ K}) (0.0348 \text{ kg/s}) (0.96483 - 0.94456) \text{ kJ/kg} \cdot \text{K} = 0.210 \text{ kW}
\]
A refrigerator with refrigerant-134a as the working fluid is considered. The rate of heat removal from the refrigerated space, the power input to the compressor, the isentropic efficiency of the compressor, and the COP of the refrigerator are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**
(a) From the refrigerant tables (Tables A-12 and A-13),

\[
\begin{align*}
P_1 & = 0.14 \text{ MPa} \quad \rightarrow \quad h_1 = 246.36 \text{ kJ/kg} \\
T_1 & = -10^\circ\text{C} \quad \rightarrow \quad \frac{s_1}{T_1} = 0.97236 \text{ kJ/kg} \cdot \text{K}
\end{align*}
\]

\[
\begin{align*}
P_2 & = 0.7 \text{ MPa} \quad \rightarrow \quad h_2 = 288.53 \text{ kJ/kg} \\
T_2 & = 50^\circ\text{C}
\end{align*}
\]

\[
\begin{align*}
P_2 & = 0.7 \text{ MPa} \quad \rightarrow \quad h_{2s} = 281.16 \text{ kJ/kg}
\end{align*}
\]

\[
\begin{align*}
P_3 & = 0.65 \text{ MPa} \quad \rightarrow \quad h_3 = h_f @ 24^\circ\text{C} = 84.98 \text{ kJ/kg}
\end{align*}
\]

\[
\begin{align*}
h_4 & = h_3 = 84.98 \text{ kJ/kg} \quad \text{(throttling)}
\end{align*}
\]

Then the rate of heat removal from the refrigerated space and the power input to the compressor are determined from

\[
\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.12 \text{ kg/s}) (246.36 - 84.98) \text{ kJ/kg} = 19.4 \text{ kW}
\]

and

\[
\dot{W}_\text{in} = \dot{m}(h_2 - h_1) = (0.12 \text{ kg/s}) (288.53 - 246.36) \text{ kJ/kg} = 5.06 \text{ kW}
\]

(b) The adiabatic efficiency of the compressor is determined from

\[
\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{281.16 - 246.36}{288.53 - 246.36} = 82.5\%
\]

(c) The COP of the refrigerator is determined from its definition,

\[
\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_\text{in}} = \frac{19.4 \text{ kW}}{5.06 \text{ kW}} = 3.83
\]
An ice-making machine operates on the ideal vapor-compression refrigeration cycle, using refrigerant-134a as the working fluid. The power input to the ice machine is to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

**Analysis** In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12E and A-13E),

\[
\begin{align*}
P_1 &= 20 \text{ psia} & h_1 &= h_g @ 20 \text{ psia} = 102.73 \text{ Btu/lbm} \\
& \text{sat. vapor} & s_1 &= s_g @ 20 \text{ psia} = 0.22567 \text{ Btu/lbm} \cdot \text{R} \\
\frac{P_2}{s_2} &= 80 \text{ psia} & h_2 &= 115.00 \text{ Btu/lbm} \\
\frac{P_3}{s_3} &= 80 \text{ psia} & h_3 &= h_f @ 80 \text{ psia} = 33.39 \text{ Btu/lbm} \\
& \text{sat. liquid} & h_4 &\approx h_3 = 33.39 \text{ Btu/lbm} \text{ (throttling)}
\end{align*}
\]

The cooling load of this refrigerator is

\[\dot{Q}_L = \dot{m}_{\text{ice}} (\Delta h)_{\text{ice}} = (15/3600 \text{ lbm/s})(169 \text{ Btu/lbm}) = 0.7042 \text{ Btu/s}\]

Then the mass flow rate of the refrigerant and the power input become

\[
\dot{m}_R = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{0.7042 \text{ Btu/s}}{102.73 - 33.39} \text{ Btu/lbm} = 0.01016 \text{ lbm/s}
\]

and

\[
\dot{W}_{\text{in}} = \dot{m}_R (h_2 - h_1) = (0.01016 \text{ lbm/s})(115.00 - 102.73) \text{ Btu/lbm} \left( \frac{1 \text{ hp}}{0.7068 \text{ Btu/s}} \right) = 0.176 \text{ hp}
\]
A refrigerator with refrigerant-134a as the working fluid is considered. The power input to the compressor, the rate of heat removal from the refrigerated space, and the pressure drop and the rate of heat gain in the line between the evaporator and the compressor are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) From the refrigerant tables (Tables A-12 and A-13),

\[
\begin{align*}
\dot{m} &= \frac{\dot{Q}_1}{v_1} = \frac{0.3/60}{0.14605} = 0.03423 \text{ kg/s} \\
\dot{W}_\text{in} &= \dot{m}(h_{2v} - h_1) = (0.03423)(289.20 - 246.36)(0.78) = 1.88 \text{ kW}
\end{align*}
\]

(b) The rate of heat removal from the refrigerated space is

\[
\dot{Q}_L = \dot{m}(h_5 - h_4) = (0.03423)(239.33 - 93.58) = 4.99 \text{ kW}
\]

(c) The pressure drop and the heat gain in the line between the evaporator and the compressor are

\[
\Delta P = P_5 - P_1 = 141.65 - 140 = 1.65
\]

and

\[
\dot{Q}_\text{gain} = \dot{m}(h_1 - h_5) = (0.03423)(246.36 - 239.33) = 0.241 \text{ kW}
\]
Problem 11-20 is reconsidered. The effects of the compressor isentropic efficiency and the compressor inlet volume flow rate on the power input and the rate of refrigeration are to be investigated.

**Analysis** The problem is solved using EES, and the solution is given below.

"Input Data"
"T[5]=-18.5 [°C]
P[1]=140 [kPa]
T[1] = -10 [°C]
V_dot[1]=0.1 [m^3/min]
P[2] = 1000 [kPa]
P[3]=950 [kPa]
T[3] = 30 [°C]
Eta_c=0.78
Fluid$='R134a'"

"Compressor"
\[ h[1]=\text{enthalpy(Fluid$\$,P=P[1],T=T[1])} \] "properties for state 1"
\[ s[1]=\text{entropy(Fluid$\$,P=P[1],T=T[1])} \] "properties for state 1"
\[ v[1]=\text{volume(Fluid$\$,P=P[1],T=T[1])} \] "[m^3/kg]"
\[ m\_dot=V\_dot[1]/v[1]\] "convert(m^3/min,m^3/s)" "[kg/s]"
\[ h_2s=\text{enthalpy(Fluid$\$,P=P[2],s=s[1])} \] "Identifies state 2s as isentropic"
\[ h[1]+Wcs=h_2s \] "energy balance on isentropic compressor"
\[ Wc=Wcs/Eta_c \] "definition of compressor isentropic efficiency"
\[ s[2]=\text{entropy(Fluid$\$,h=h[2],P=P[2])} \] "properties for state 2" 
\[ T[2]=\text{temperature(Fluid$\$,h=h[2],P=P[2])} \]
\[ W\_dot\_c=m\_dot*Wc \]

"Condenser"
\[ h[3]=\text{enthalpy(Fluid$\$,P=P[3],T=T[3])} \] "properties for state 3"
\[ s[3]=\text{entropy(Fluid$\$,P=P[3],T=T[3])} \]
\[ h[2]=q\_out+h[3] \] "energy balance on condenser"
\[ Q\_dot\_out=m\_dot*q\_out \]

"Throttle Valve"
\[ x[4]=\text{quality(Fluid$\$,h=h[4],P=P[4])} \] "properties for state 4"
\[ s[4]=\text{entropy(Fluid$\$,h=h[4],P=P[4])} \]
\[ T[4]=\text{temperature(Fluid$\$,h=h[4],P=P[4])} \]

"Evaporator"
\[ P[4]=\text{pressure(Fluid$\$,T=T[5],x=0)} \] "pressure=Psat at evaporator exit temp."
\[ h[5]=\text{enthalpy(Fluid$\$,T=T[5],x=1)} \] "properties for state 5"
\[ Q\_dot\_in=m\_dot*q\_in \]
\[ COP=Q\_dot\_in/W\_dot\_c \] "definition of COP"
\[ COP\_plot = COP \]
\[ W\_dot\_in = W\_dot\_c \]
\[ Q\_dot\_line5to1=m\_dot*(h[1]-h[5])\] "[kW]"
<table>
<thead>
<tr>
<th>(\text{COP}_{\text{plot}})</th>
<th>(W_{\text{h}}) [kW]</th>
<th>(Q_{\text{h}}) [kW]</th>
<th>(\eta_{c}) [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.041</td>
<td>0.8149</td>
<td>1.663</td>
<td>0.6</td>
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<tr>
<td>2.381</td>
<td>0.6985</td>
<td>1.663</td>
<td>0.7</td>
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<td>2.721</td>
<td>0.6112</td>
<td>1.663</td>
<td>0.8</td>
</tr>
<tr>
<td>3.062</td>
<td>0.5433</td>
<td>1.663</td>
<td>0.9</td>
</tr>
<tr>
<td>3.402</td>
<td>0.4889</td>
<td>1.663</td>
<td>1</td>
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</tbody>
</table>
A refrigerator uses refrigerant-134a as the working fluid and operates on the ideal vapor-compression refrigeration cycle. The mass flow rate of the refrigerant, the condenser pressure, and the COP of the refrigerator are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) (b) From the refrigerant-134a tables (Tables A-11 through A-13)

\[
\begin{align*}
P_4 &= 120 \text{kPa} \\
x_4 &= 0.3 \\
h_4 &= 86.83 \text{kJ/kg} \\
h_3 &= h_4 \\
h_3 &= 86.83 \text{kJ/kg} \\
x_3 &= 0 \text{ (sat. liq.)} \\
P_3 &= h_3 \text{ kPa} = 671.8 \text{kPa}
\end{align*}
\]

The mass flow rate of the refrigerant is determined from

\[
\dot{m} = \frac{\dot{W}_{in}}{h_2 - h_1} = \frac{0.45 \text{kW}}{(298.87 - 236.97) \text{kJ/kg}} = 0.00727 \text{ kg/s}
\]

(c) The refrigeration load and the COP are

\[
\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.0727 \text{ kg/s})(236.97 - 86.83) \text{kJ/kg} = 1.091 \text{kW}
\]

\[
\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{1.091 \text{kW}}{0.45 \text{kW}} = 2.43
\]
Selecting the Right Refrigerant

11-23C The desirable characteristics of a refrigerant are to have an evaporator pressure which is above the atmospheric pressure, and a condenser pressure which corresponds to a saturation temperature above the temperature of the cooling medium. Other desirable characteristics of a refrigerant include being nontoxic, noncorrosive, nonflammmable, chemically stable, having a high enthalpy of vaporization (minimizes the mass flow rate) and, of course, being available at low cost.

11-24C The minimum pressure that the refrigerant needs to be compressed to is the saturation pressure of the refrigerant at 30°C, which is 0.771 MPa. At lower pressures, the refrigerant will have to condense at temperatures lower than the temperature of the surroundings, which cannot happen.

11-25C Allowing a temperature difference of 10°C for effective heat transfer, the evaporation temperature of the refrigerant should be -20°C. The saturation pressure corresponding to -20°C is 0.133 MPa. Therefore, the recommended pressure would be 0.12 MPa.

11-26 A refrigerator that operates on the ideal vapor-compression cycle with refrigerant-134a is considered. Reasonable pressures for the evaporator and the condenser are to be selected.

Assumptions 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible.

Analysis Allowing a temperature difference of 10°C for effective heat transfer, the evaporation and condensation temperatures of the refrigerant should be -20°C and 35°C, respectively. The saturation pressures corresponding to these temperatures are 0.133 MPa and 0.888 MPa. Therefore, the recommended evaporator and condenser pressures are 0.133 MPa and 0.888 MPa, respectively.

Heat Pump Systems

11-28C A heat pump system is more cost effective in Miami because of the low heating loads and high cooling loads at that location.

11-29C A water-source heat pump extracts heat from water instead of air. Water-source heat pumps have higher COPs than the air-source systems because the temperature of water is higher than the temperature of air in winter.
A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a is considered. The power input to the heat pump and the electric power saved by using a heat pump instead of a resistance heater are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**
In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12E and A-13E),

\[
P_1 = 50 \text{ psia} \quad h_1 = h_g @ 50 \text{ psia} = 108.81 \text{ Btu/lbm}
\]

sat. vapor

\[
S_1 = S_g @ 50 \text{ psia} = 0.22188 \text{ Btu/lbm} \cdot \text{R}
\]

\[
P_2 = 120 \text{ psia} \quad h_2 = 116.62 \text{ Btu/lbm}
\]

\[
S_2 = S_1
\]

\[
P_3 = 120 \text{ psia} \quad h_3 = h_f @ 120 \text{ psia} = 41.79 \text{ Btu/lbm}
\]

sat. liquid

\[
h_4 \approx h_3 = 41.79 \text{ Btu/lbm} \text{ (throttling)}
\]

The mass flow rate of the refrigerant and the power input to the compressor are determined from

\[
\dot{m} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{60,000/3600 \text{ Btu/s}}{(116.62 - 41.79) \text{ Btu/lbm}} = 0.2227 \text{ lbm/s}
\]

and

\[
\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.2227 \text{ kg/s})(116.62 - 108.81) \text{ Btu/lbm}
\]

\[
= 1.738 \text{ Btu/s} = 2.46 \text{ hp since } 1 \text{ hp} = 0.7068 \text{ Btu/s}
\]

The electrical power required without the heat pump is

\[
\dot{W}_e = \dot{Q}_H = (60,000/3600 \text{ Btu/s}) \left(\frac{1 \text{ hp}}{0.7068 \text{ Btu/s}}\right) = 23.58 \text{ hp}
\]

Thus,

\[
\dot{W}_{\text{saved}} = \dot{W}_e - \dot{W}_{in} = 23.58 - 2.46
\]

\[
= 21.1 \text{ hp} = 15.75 \text{ kW since } 1 \text{ hp} = 0.7457 \text{ kW}
\]
A heat pump that operates on the ideal vapor-compression cycle with refrigerant-134a is considered. The power input to the heat pump is to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**
In an ideal vapor-compression refrigeration cycle, the compression process is isentropic, the refrigerant enters the compressor as a saturated vapor at the evaporator pressure, and leaves the condenser as saturated liquid at the condenser pressure. From the refrigerant tables (Tables A-12 and A-13),

\[
P_1 = 320 \text{ kPa} \quad h_1 = h_g @ 320 \text{ kPa} = 251.88 \text{ kJ/kg}
\]

sat. vapor \quad \int s_1 = s_g @ 320 \text{ kPa} = 0.93006 \text{ kJ/kg \cdot K}

\[
P_2 = 1.4 \text{ MPa} \quad h_2 = 282.54 \text{ kJ/kg}
\]

\[
s_2 = s_1
\]

\[
P_3 = 1.4 \text{ MPa} \quad h_3 = h_f @ 1.4 \text{ MPa} = 127.22 \text{ kJ/kg}
\]

sat. liquid \quad h_4 \cong h_3 = 127.22 \text{ kJ/kg} \quad \text{(throttling)}

The heating load of this heat pump is determined from

\[
\dot{Q}_H = \dot{m}_c (T_2 - T_1) \quad \text{water}
\]

\[
= (0.12 \text{ kg/s})(4.18 \text{ kJ/kg \cdot ^\circ C})(45 - 15) ^\circ C = 15.05 \text{ kW}
\]

and

\[
\dot{m}_R = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{15.05 \text{ kJ/s}}{(282.54 - 127.22) \text{ kJ/kg}} = 0.09688 \text{ kg/s}
\]

Then,

\[
W_{in} = \dot{m}_R (h_2 - h_1) = (0.09688 \text{ kg/s})(282.54 - 251.88) \text{ kJ/kg} = 2.97 \text{ kW}
\]
A heat pump with refrigerant-134a as the working fluid heats a house by using underground water as the heat source. The power input to the heat pump, the rate of heat absorption from the water, and the increase in electric power input if an electric resistance heater is used instead of a heat pump are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) From the refrigerant tables (Tables A-12 and A-13),

\[
\begin{align*}
P_1 &= 280 \text{kPa} \\
T_1 &= 0^\circ \text{C} \quad h_1 = 250.83 \text{ kJ/kg} \\

P_2 &= 1.0 \text{ MPa} \\
T_2 &= 60^\circ \text{C} \quad h_2 = 293.38 \text{ kJ/kg} \\

P_3 &= 1.0 \text{ MPa} \\
T_3 &= 30^\circ \text{C} \quad h_3 = 93.58 \text{ kJ/kg} \\

h_4 &\approx h_3 = 93.58 \text{ kJ/kg} \quad \text{(throttling)}
\end{align*}
\]

The mass flow rate of the refrigerant is

\[
\dot{m}_R = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{h_2 - h_3} = \frac{60,000/3,600 \text{ kJ/s}}{(293.38 - 93.58) \text{ kJ/kg}} = 0.08341 \text{ kg/s}
\]

Then the power input to the compressor becomes

\[
\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.08341 \text{ kg/s})(293.38 - 250.83) \text{ kJ/kg} = 3.55 \text{ kW}
\]

(b) The rate of heat absorption from the water is

\[
\dot{Q}_L = \dot{m}(h_4 - h_1) = (0.08341 \text{ kg/s})(250.83 - 93.58) \text{ kJ/kg} = 13.12 \text{ kW}
\]

(c) The electrical power required without the heat pump is

\[
\dot{W}_e = \dot{Q}_H = 60,000/3600 \text{ kJ/s} = 16.67 \text{ kW}
\]

Thus,

\[
\dot{W}_{increase} = \dot{W}_e - \dot{W}_{in} = 16.67 - 3.55 = 13.12 \text{ kW}
\]
11-32 EES Problem 11-32 is reconsidered. The effect of the compressor isentropic efficiency on the power input to the compressor and the electric power saved by using a heat pump rather than electric resistance heating is to be investigated.

**Analysis** The problem is solved using EES, and the solution is given below.

"**Input Data**"  
"**Input Data is supplied in the diagram window**"  
"P[1]=280 [kPa]  
T[1] = 0 [C]  
P[2] = 1000 [kPa]  
T[3] = 30 [C]  
Q dot out = 60000 [kJ/h]  
Eta_c=1.0  
Fluid$="R134a"  
"Use ETA_c = 0.623 to obtain T[2] = 60C"

"**Compressor**"  
h[1]=enthalpy(Fluid$,P=P[1],T=T[1])  "properties for state 1"  
s[1]=entropy(Fluid$,P=P[1],T=T[1])  
h2s=enthalpy(Fluid$,P=P[2],s=s[1])  "Identifies state 2s as isentropic"  
h[1]+Wcs=h2s  "energy balance on isentropic compressor"  
Wc=Wcs/Eta_c  "definition of compressor isentropic efficiency"  
s[2]=entropy(Fluid$,h=h[2],P=P[2])  "properties for state 2"  
{h[2]=enthalpy(Fluid$,P=P[2],T=T[2]) }  
T[2]=temperature(Fluid$,h=h[2],P=P[2])  
W dot c=m dot*Wc

"**Condenser**"  
h[3]=enthalpy(Fluid$,P=P[3],T=T[3])  "properties for state 3"  
s[3]=entropy(Fluid$,P=P[3],T=T[3])  
Q dot out*convert(kJ/h,kJ/s)=m dot*Qout

"**Throttle Valve**"  
x[4]=quality(Fluid$,h=h[4],P=P[4])  "properties for state 4"  
s[4]=entropy(Fluid$,h=h[4],P=P[4])  
T[4]=temperature(Fluid$,h=h[4],P=P[4])

"**Evaporator**"  
Q dot in=m dot*Q in  
COP=Q dot out*convert(kJ/h,kJ/s)/W dot c  "definition of COP"  
COP_plot = COP  
W dot in = W dot c  
E dot saved = Q dot out*convert(kJ/h,kJ/s) - W dot c[kW]"
<table>
<thead>
<tr>
<th>$W_n$ [kW]</th>
<th>$\eta_c$</th>
<th>$E_{saved}$</th>
</tr>
</thead>
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<tr>
<td>3.671</td>
<td>0.6</td>
<td>13</td>
</tr>
<tr>
<td>3.249</td>
<td>0.7</td>
<td>13.42</td>
</tr>
<tr>
<td>2.914</td>
<td>0.8</td>
<td>13.75</td>
</tr>
<tr>
<td>2.641</td>
<td>0.9</td>
<td>14.03</td>
</tr>
<tr>
<td>2.415</td>
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<td>14.25</td>
</tr>
</tbody>
</table>
An actual heat pump cycle with R-134a as the refrigerant is considered. The isentropic efficiency of the compressor, the rate of heat supplied to the heated room, the COP of the heat pump, and the COP and the rate of heat supplied to the heated room if this heat pump operated on the ideal vapor-compression cycle between the same pressure limits are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) The properties of refrigerant-134a are (Tables A-11 through A-13)

\[
\begin{align*}
T_2 &= 55^\circ C \\
P_2 &= 800 \text{ kPa} \\
h_2 &= 291.76 \text{ kJ/kg}
\end{align*}
\]

\[
\begin{align*}
T_3 &= T_{\text{sat}}(750 \text{ kPa}) = 29.06^\circ C \\
P_3 &= 750 \text{ kPa} \\
h_3 &= 87.91 \text{ kJ/kg}
\end{align*}
\]

\[
\begin{align*}
h_4 &= h_3 = 87.91 \text{ kJ/kg} \\
P_{\text{sat}}(200 \text{ kPa}) &= -10.09^\circ C \\
P_1 &= 200 \text{ kPa} \\
h_1 &= 247.87 \text{ kJ/kg} \\
T_1 &= (-10.09 + 4)^\circ C = 0.9506 \text{ kJ/kg} \\
P_2 &= 800 \text{ kPa} \\
h_2 &= h_{s2} = 277.26
\end{align*}
\]

The isentropic efficiency of the compressor is

\[
\eta_C = \frac{h_{s2} - h_1}{h_2 - h_1} = \frac{277.26 - 247.87}{291.76 - 247.87} = 0.670
\]

(b) The rate of heat supplied to the room is

\[
\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.018 \text{ kg/s})(291.76 - 87.91) \text{ kJ/kg} = 3.67 \text{ kW}
\]

(c) The power input and the COP are

\[
\dot{W}_\text{in} = \dot{m}(h_2 - h_1) = (0.018 \text{ kg/s})(291.76 - 247.87) \text{ kJ/kg} = 0.790 \text{ kW}
\]

\[
\text{COP} = \frac{\dot{Q}_H}{\dot{W}_\text{in}} = \frac{3.67}{0.790} = 4.64
\]

(d) The ideal vapor-compression cycle analysis of the cycle is as follows:

\[
\begin{align*}
h_1 &= h_{s1}(200 \text{ kPa}) = 244.46 \text{ kJ/kg} \\
s_1 &= s_{g1}(200 \text{ kPa}) = 0.9377 \text{ kJ/kg.K} \\
P_2 &= 800 \text{ MPa} \\
h_2 &= h_{s2} = 273.25 \text{ kJ/kg} \\
s_2 &= s_1 \\
h_3 &= h_{f}(800 \text{ kPa}) = 95.47 \text{ kJ/kg} \\
h_4 &= h_3
\end{align*}
\]

\[
\text{COP} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{273.25 - 95.47}{273.25 - 244.46} = 6.18
\]

\[
\dot{Q}_H = \dot{m}(h_2 - h_3) = (0.018 \text{ kg/s})(273.25 - 95.47) \text{ kJ/kg} = 3.20 \text{ kW}
\]
A geothermal heat pump is considered. The degrees of subcooling done on the refrigerant in the condenser, the mass flow rate of the refrigerant, the heating load, the COP of the heat pump, the minimum power input are to be determined.

**Assumptions**

1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) From the refrigerant-134a tables (Tables A-11 through A-13)

\[
\begin{align*}
T_4 &= 20\degree C \quad P_4 = 572.1\text{kPa} \\
x_4 &= 0.23 \quad h_4 = 121.24\text{kJ/kg} \\
h_3 &= h_4 \\
P_1 &= 572.1\text{kPa} \quad h_1 = 261.59\text{kJ/kg} \\
x_1 &= 1\text{(sat. vap.)} \quad s_1 = 0.9223\text{kJ/kg} \\
P_2 &= 1400\text{kPa} \quad h_2 = 280.00\text{kJ/kg} \\
s_2 &= s_1
\end{align*}
\]

From the steam tables (Table A-4)

\[
\begin{align*}
\dot{h}_{w1} &= h_f(50\degree C) = 209.34\text{kJ/kg} \\
\dot{h}_{w2} &= h_f(40\degree C) = 167.53\text{kJ/kg}
\end{align*}
\]

The saturation temperature at the condenser pressure of 1400 kPa and the actual temperature at the condenser outlet are

\[
T_{\text{sat}}(1400\text{kPa}) = 52.40\degree C \\
P_3 = 1400\text{kPa} \quad h_3 = 121.24\text{kJ/kg} \\
T_3 = 48.59\degree C \quad \text{(from EES)}
\]

Then, the degrees of subcooling is

\[
\Delta T_{\text{subcool}} = T_{\text{sat}} - T_3 = 52.40 - 48.59 = 3.81\degree C
\]

(b) The rate of heat absorbed from the geothermal water in the evaporator is

\[
\dot{Q}_L = \dot{m}_w(h_{\text{sat}} - h_{w2}) = (0.065\text{kg/s})(209.34 - 167.53)\text{kJ/kg} = 2.718\text{kw}
\]

This heat is absorbed by the refrigerant in the evaporator

\[
\dot{m}_R = \frac{\dot{Q}_L}{h_1 - h_4} = \frac{2.718\text{kw}}{(261.59 - 121.24)\text{kJ/kg}} = 0.01936\text{kg/s}
\]

(c) The power input to the compressor, the heating load and the COP are

\[
\begin{align*}
\dot{W}_{\text{in}} &= \dot{m}_R(h_2 - h_1) + \dot{Q}_{\text{out}} = (0.01936\text{kg/s})(280.00 - 261.59)\text{kJ/kg} = 0.6564\text{kw} \\
\dot{Q}_H &= \dot{m}_R(h_2 - h_3) = (0.01936\text{kg/s})(280.00 - 121.24)\text{kJ/kg} = 3.074\text{kw} \\
\text{COP} &= \frac{\dot{Q}_H}{\dot{W}_{\text{in}}} = \frac{3.074\text{kw}}{0.6564\text{kw}} = 4.68
\end{align*}
\]

(d) The reversible COP of the cycle is

\[
\text{COP}_{\text{rev}} = \frac{1}{1 - \frac{T_L}{T_H}} = \frac{1}{1 - (25 + 273)/(50 + 273)} = 12.92
\]

The corresponding minimum power input is

\[
\dot{W}_{\text{in, min}} = \frac{\dot{Q}_H}{\text{COP}_{\text{rev}}} = \frac{3.074\text{kw}}{12.92} = 0.238\text{kw}
\]
Innovative Refrigeration Systems

11-36C Performing the refrigeration in stages is called cascade refrigeration. In cascade refrigeration, two or more refrigeration cycles operate in series. Cascade refrigerators are more complex and expensive, but they have higher COPs, they can incorporate two or more different refrigerants, and they can achieve much lower temperatures.

11-37C Cascade refrigeration systems have higher COPs than the ordinary refrigeration systems operating between the same pressure limits.

11-38C The saturation pressure of refrigerant-134a at -32°C is 77 kPa, which is below the atmospheric pressure. In reality a pressure below this value should be used. Therefore, a cascade refrigeration system with a different refrigerant at the bottoming cycle is recommended in this case.

11-39C We would favor the two-stage compression refrigeration system with a flash chamber since it is simpler, cheaper, and has better heat transfer characteristics.

11-40C Yes, by expanding the refrigerant in stages in several throttling devices.

11-41C To take advantage of the cooling effect by throttling from high pressures to low pressures.
A two-stage cascade refrigeration system is considered. Each stage operates on the ideal vapor-compression cycle with refrigerant-134a as the working fluid. The mass flow rate of refrigerant through the lower cycle, the rate of heat removal from the refrigerated space, the power input to the compressor, and the COP of this cascade refrigerator are to be determined.

Assumptions
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.
3. The heat exchanger is adiabatic.

Analysis
(a) Each stage of the cascade refrigeration cycle is said to operate on the ideal vapor compression refrigeration cycle. Thus the compression process is isentropic, and the refrigerant enters the compressor as a saturated vapor at the evaporator pressure. Also, the refrigerant leaves the condenser as a saturated liquid at the condenser pressure. The enthalpies of the refrigerant at all 8 states are determined from the refrigerant tables (Tables A-11, A-12, and A-13) to be

\[
\begin{align*}
    h_1 &= 239.16 \text{ kJ/kg}, \quad h_2 = 260.58 \text{ kJ/kg} \\
    h_3 &= 63.94 \text{ kJ/kg}, \quad h_4 = 63.94 \text{ kJ/kg} \\
    h_5 &= 255.55 \text{ kJ/kg}, \quad h_6 = 269.91 \text{ kJ/kg} \\
    h_7 &= 95.47 \text{ kJ/kg}, \quad h_8 = 94.63 \text{ kJ/kg} \\
\end{align*}
\]

The mass flow rate of the refrigerant through the lower cycle is determined from an energy balance on the heat exchanger:

\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta E_{\text{system}} \overset{\text{steady}}{=} 0
\]

\[
\dot{E}_{\text{in}} = \dot{E}_{\text{out}}
\]

\[
\sum \dot{m}_i h_i = \dot{m}_{\text{in}} h_{\text{in}} = \dot{m}_A (h_5 - h_8) = \dot{m}_B (h_2 - h_3)
\]

\[
\dot{m}_B = \frac{h_5 - h_8}{h_2 - h_3}
\]

\[
\dot{m}_A = \frac{255.55 - 95.47}{260.58 - 63.94} (0.24 \text{ kg/s}) = 0.1954 \text{ kg/s}
\]

(b) The rate of heat removed by a cascade cycle is the rate of heat absorption in the evaporator of the lowest stage. The power input to a cascade cycle is the sum of the power inputs to all of the compressors:

\[
\dot{Q}_L = \dot{m}_B (h_1 - h_4) = (0.1954 \text{ kg/s})(239.16 - 63.94) \text{ kJ/kg} = 34.24 \text{ kW}
\]

\[
\dot{W}_{\text{in}} = \dot{W}_{\text{comp}, \text{in}} + \dot{W}_{\text{comp}, \text{in}} = \dot{m}_A (h_6 - h_5) + \dot{m}_B (h_2 - h_1)
\]

\[
= (0.24 \text{ kg/s})(269.91 - 255.55) \text{ kJ/kg} + (0.1954 \text{ kg/s})(260.58 - 239.16) \text{ kJ/kg}
\]

\[
= 7.63 \text{ kW}
\]

(c) The COP of this refrigeration system is determined from its definition,

\[
\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net}, \text{in}}} = \frac{34.24 \text{ kW}}{7.63 \text{ kW}} = 4.49
\]
A two-stage cascade refrigeration system is considered. Each stage operates on the ideal vapor-compression cycle with refrigerant-134a as the working fluid. The mass flow rate of refrigerant through the lower cycle, the rate of heat removal from the refrigerated space, the power input to the compressor, and the COP of this cascade refrigerator are to be determined.

**Assumptions**

1. Steady operating conditions exist.  
2. Kinetic and potential energy changes are negligible.  
3. The heat exchanger is adiabatic.

**Analysis**

(a) Each stage of the cascade refrigeration cycle is said to operate on the ideal vapor compression refrigeration cycle. Thus the compression process is isentropic, and the refrigerant enters the compressor as a saturated vapor at the evaporator pressure. Also, the refrigerant leaves the condenser as a saturated liquid at the condenser pressure. The enthalpies of the refrigerant at all 8 states are determined from the refrigerant tables (Tables A-11, A-12, and A-13) to be

\[
\begin{align*}
  h_1 &= 239.16 \text{ kJ/kg} , \\
  h_2 &= 267.34 \text{ kJ/kg} , \\
  h_3 &= 77.54 \text{ kJ/kg} , \\
  h_4 &= 77.54 \text{ kJ/kg} , \\
  h_5 &= 260.92 \text{ kJ/kg} , \\
  h_6 &= 268.66 \text{ kJ/kg} , \\
  h_7 &= 95.47 \text{ kJ/kg} , \\
  h_8 &= 95.47 \text{ kJ/kg} .
\end{align*}
\]

The mass flow rate of the refrigerant through the lower cycle is determined from an energy balance on the heat exchanger:

\[
\hat{E}_{in} - \hat{E}_{out} = \Delta \hat{E}_{system} \neq \hat{E}_{system} (\text{steady}) = 0
\]

\[
\hat{E}_{in} = \hat{E}_{out}
\]

\[
\sum m_i h_i = \sum m_i h_i
\]

\[
m_A(h_5 - h_8) = m_B(h_2 - h_1)
\]

\[
m_B = \frac{h_5 - h_8}{h_2 - h_1}
\]

\[
m_A = \frac{260.92 - 95.47}{267.34 - 77.54} (0.24 \text{ kg/s}) = 0.2092 \text{ kg/s}
\]

(b) The rate of heat removed by a cascade cycle is the rate of heat absorption in the evaporator of the lowest stage. The power input to a cascade cycle is the sum of the power inputs to all of the compressors:

\[
\dot{Q}_L = m_A(h_1 - h_4) = (0.2092 \text{ kg/s}) (239.16 - 77.54) \text{ kJ/kg} = 33.81 \text{ kW}
\]

\[
\dot{W}_{in} = \dot{W}_{\text{comp, in}} + \dot{W}_{\text{comp, out}} = m_A(h_6 - h_5) + m_B(h_2 - h_1)
\]

\[
= (0.24 \text{ kg/s}) (268.66 - 260.92) \text{ kJ/kg} + (0.2092 \text{ kg/s}) (267.34 - 239.16) \text{ kJ/kg}
\]

\[
= 7.75 \text{ kW}
\]

(c) The COP of this refrigeration system is determined from its definition,

\[
\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net, in}}} = \frac{33.81 \text{ kW}}{7.75 \text{ kW}} = 4.36
\]
A two-stage compression refrigeration system with refrigerant-134a as the working fluid is considered. The fraction of the refrigerant that evaporates as it is throttled to the flash chamber, the rate of heat removed from the refrigerated space, and the COP are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.
3. The flash chamber is adiabatic.

**Analysis**

(a) The enthalpies of the refrigerant at several states are determined from the refrigerant tables (Tables A-11, A-12, and A-13) to be

\[ h_1 = 239.16 \text{ kJ/kg}, \quad h_2 = 265.31 \text{ kJ/kg} \]
\[ h_3 = 259.30 \text{ kJ/kg}, \quad h_4 = 107.32 \text{ kJ/kg} \]
\[ h_5 = 107.32 \text{ kJ/kg}, \quad h_6 = 107.32 \text{ kJ/kg} \]
\[ h_7 = 73.33 \text{ kJ/kg}, \quad h_8 = 73.33 \text{ kJ/kg} \]

The fraction of the refrigerant that evaporates as it is throttled to the flash chamber is simply the quality at state 6,

\[ x_6 = \frac{h_6 - h_f}{h_{fg}} = \frac{107.32 - 73.33}{185.98} = 0.1828 \]

(b) The enthalpy at state 9 is determined from an energy balance on the mixing chamber:

\[ \dot{E}_\text{in} - \dot{E}_\text{out} = \Delta \dot{E}_\text{system} \]

\[ \sum \dot{m}_i h_i = \dot{E}_\text{out} + \sum \dot{m}_i h_i \]

\[ (1)h_9 = x_6 h_5 + (1 - x_6)h_2 \]
\[ h_9 = (0.1828)(259.30) + (1 - 0.1828)(265.31) = 264.21 \text{ kJ/kg} \]

also,

\[ P_4 = 1 \text{ MPa} \]
\[ s_4 = s_8 = 0.94083 \text{ kJ/kg} \cdot \text{K} \]

\[ h_4 = 278.97 \text{ kJ/kg} \]

Then the rate of heat removed from the refrigerated space and the compressor work input per unit mass of refrigerant flowing through the condenser are

\[ \dot{m}_B = (1 - x_4) \dot{m}_A = (1 - 0.1828)(0.25 \text{ kg/s}) = 0.2043 \text{ kg/s} \]

\[ \dot{Q}_L = \dot{m}_B (h_4 - h_8) = (0.2043 \text{ kg/s})(239.16 - 73.33) \text{ kJ/kg} = 33.88 \text{ kW} \]

\[ \dot{W}_\text{in} = \dot{W}_\text{compl,In} + \dot{W}_\text{compl,In} = \dot{m}_A (h_4 - h_9) + \dot{m}_B (h_2 - h_1) \]
\[ = (0.25 \text{ kg/s})(278.97 - 264.21) \text{ kJ/kg} + (0.2043 \text{ kg/s})(265.31 - 239.16) \text{ kJ/kg} \]
\[ = 9.03 \text{ kW} \]

(c) The coefficient of performance is determined from

\[ \text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net,in}}} = \frac{33.88 \text{ kW}}{9.03 \text{ kW}} = 3.75 \]
Problem 11-44 is reconsidered. The effects of the various refrigerants in EES data bank for compressor efficiencies of 80, 90, and 100 percent is to be investigated.

**Analysis** The problem is solved using EES, and the results are tabulated and plotted below.

"Input Data"
*P[1]=140 [kPa]*
*P[4]= 1000 [kPa]*
*P[6]=500 [kPa]*
*Eta_compB =1.0*
*Eta_compA =1.0*
*m_dot_A=0.25 [kg/s]*

"High Pressure Compressor A"

h4s=enthalpy(R134a,P=P[4],s=s[9]) "State 4s is the isentropic value of state 4"
h[9]+w_compAs=h4s "energy balance on isentropic compressor"
w_compA=w_compAs/Eta_compA "definition of compressor isentropic efficiency"
s[4]=entropy(R134a,h=h[4],P=P[4]) "properties for state 4"
T[4]=temperature(R134a,h=h[4],P=P[4])
W_dot_compA=m_dot_A*w_compA

"Condenser"

T[5]=temperature(R134a,h,h[5]=0) "properties for state 5, assumes sat. liq. at cond. exit"
h[5]=enthalpy(R134a,T=T[5],x=0) "properties for state 5"
s[5]=entropy(R134a,T=T[5],x=0)
Q_dot_out = m_dot_A*q_out

"Throttle Valve A"

x6=quality(R134a,h=h[6],P=P[6]) "properties for state 6"
s[6]=entropy(R134a,h=h[6],P=P[6])
T[6]=temperature(R134a,h=h[6],P=P[6])

"Flash Chamber"

m_dot_B = (1-x6) * m_dot_A
h[7]=enthalpy(R134a, P=P[7], x=0)
s[7]=entropy(R134a,h=h[7],P=P[7])
T[7]=temperature(R134a,h=h[7],P=P[7])

"Mixing Chamber"

x6*m_dot_A*h[3] + m_dot_B*h[2] =(x6* m_dot_A + m_dot_B)*h[9]
h[3]=enthalpy(R134a, P=P[3], x=1) "properties for state 3"
s[3]=entropy(R134a,P=P[3],x=1)
T[3]=temperature(R134a,P=P[3],x=x1)
s[9]=entropy(R134a,h=h[9],P=P[9]) "properties for state 9"
T[9]=temperature(R134a,h=h[9],P=P[9])

"Low Pressure Compressor B"

x1=1 "assume flow to compressor inlet to be saturated vapor"
h[1]=enthalpy(R134a,P=P[1],x=x1) "properties for state 1"
\[ T[1] = \text{temperature}(R134a, P=P[1], x=x1) \]
\[ s[1] = \text{entropy}(R134a, P=P[1], x=x1) \]
\[ h2s = \text{enthalpy}(R134a, P=P[2], s=s[1]) \]  
"state 2s is isentropic state at comp. exit"
\[ h[1] + w_{\text{compB}} = h2s \]  
"energy balance on isentropic compressor"
\[ w_{\text{compB}} = \frac{w_{\text{compB}}}{\text{Eta}_{\text{compB}}} \]  
"definition of compressor isentropic efficiency"
\[ h[1] + w_{\text{compB}} = h[2] \]  
"energy balance on real compressor-assumed adiabatic"
\[ s[2] = \text{entropy}(R134a, h=h[2], P=P[2]) \]  
"properties for state 2"
\[ T[2] = \text{temperature}(R134a, h=h[2], P=P[2]) \]
\[ W_{\text{dot compB}} = m_{\text{dot B}} \cdot w_{\text{compB}} \]

"Throttle Valve B"
\[ h[8] = h[7] \]  
"energy balance on throttle - isenthalpic"
\[ x8 = \text{quality}(R134a, h=h[8], P=P[8]) \]  
"properties for state 8"
\[ s[8] = \text{entropy}(R134a, h=h[8], P=P[8]) \]
\[ T[8] = \text{temperature}(R134a, h=h[8], P=P[8]) \]

"Evaporator"
\[ P[8] = P[1] \]  
"neglect pressure drop across evaporator"
\[ q_{\text{in}} + h[8] = h[1] \]  
"energy balance on evaporator"
\[ Q_{\text{dot in}} = m_{\text{dot B}} \cdot q_{\text{in}} \]

"Cycle Statistics"
\[ W_{\text{dot in total}} = W_{\text{dot compA}} + W_{\text{dot compB}} \]
\[ \text{COP} = \frac{Q_{\text{dot in}}}{W_{\text{dot in total}}} \]  
"definition of COP"

<table>
<thead>
<tr>
<th>( \eta_{\text{compA}} )</th>
<th>( \eta_{\text{compB}} )</th>
<th>( Q_{\text{out}} )</th>
<th>COP</th>
</tr>
</thead>
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<tr>
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<td>0.8</td>
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<td>2.963</td>
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<td>0.8333</td>
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<td>3.094</td>
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<td>0.8667</td>
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<td>3.225</td>
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<td>0.9</td>
<td>43.97</td>
<td>3.357</td>
</tr>
<tr>
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<td>0.9333</td>
<td>43.59</td>
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<tr>
<td>0.9667</td>
<td>0.9667</td>
<td>43.24</td>
<td>3.619</td>
</tr>
<tr>
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<td>1</td>
<td>42.91</td>
<td>3.751</td>
</tr>
</tbody>
</table>

\[ \text{R134a} \]
A two-stage compression refrigeration system with refrigerant-134a as the working fluid is considered. The fraction of the refrigerant that evaporates as it is throttled to the flash chamber, the rate of heat removed from the refrigerated space, and the COP are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible. 3 The flash chamber is adiabatic.

**Analysis**

(a) The enthalpies of the refrigerant at several states are determined from the refrigerant tables (Tables A-11, A-12, and A-13) to be

- \( h_1 = 239.16 \text{ kJ/kg} \)
- \( h_2 = 255.90 \text{ kJ/kg} \)
- \( h_3 = 251.88 \text{ kJ/kg} \)
- \( h_4 = 107.32 \text{ kJ/kg} \)
- \( h_5 = 107.32 \text{ kJ/kg} \)
- \( h_6 = 55.16 \text{ kJ/kg} \)
- \( h_9 = 55.16 \text{ kJ/kg} \)

The fraction of the refrigerant that evaporates as it is throttled to the flash chamber is simply the quality at state 6,

\[
x_6 = \frac{h_6 - h_f}{h_{fg}} = \frac{107.32 - 55.16}{196.71} = 0.2651
\]

(b) The enthalpy at state 9 is determined from an energy balance on the mixing chamber:

\[
\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system} \quad \text{\(\phi_0\) (steady) = 0}
\]

\[
\dot{E}_{in} = \dot{E}_{out} = \sum m_i h_i = \sum m_i h_i
\]

\[
(1)h_9 = x_6 h_3 + (1 - x_6) h_2
\]

\[
h_9 = (0.2651)(251.88) + (1 - 0.2651)(255.90) = 254.84 \text{ kJ/kg}
\]

and

\[
P_9 = 0.32 \text{ MPa}
\]

\[
h_9 = 254.84 \text{ kJ/kg}
\]

\[
s_9 = 0.94074 \text{ kJ/kg} \cdot \text{K}
\]

\[
h_4 = 278.94 \text{ kJ/kg}
\]

Then the rate of heat removed from the refrigerated space and the compressor work input per unit mass of refrigerant flowing through the condenser are

\[
\dot{m}_b = (1 - x_6) \dot{m}_a = (1 - 0.2651)(0.25 \text{ kg/s}) = 0.1837 \text{ kg/s}
\]

\[
\dot{Q}_L = \dot{m}_b (h_1 - h_9) = (0.1837 \text{ kg/s})(239.16 - 55.16) \text{ kJ/kg} = \underline{33.80} \text{ kW}
\]

\[
W_{in} = \dot{W}_{comp,in} + \dot{W}_{comp,II,in} = \dot{m}_a (h_4 - h_9) + \dot{m}_b (h_2 - h_1)
\]

\[
= (0.25 \text{ kg/s})(278.94 - 254.84) \text{ kJ/kg} + (0.1837 \text{ kg/s})(255.90 - 239.16) \text{ kJ/kg}
\]

\[
= 9.10 \text{ kW}
\]

(c) The coefficient of performance is determined from

\[
\text{COP}_R = \frac{\dot{Q}_L}{W_{net,in}} = \frac{33.80 \text{ kW}}{9.10 \text{ kW}} = 3.71
\]
A two-stage cascade refrigeration cycle is considered. The mass flow rate of the refrigerant through the upper cycle, the rate of heat removal from the refrigerated space, and the COP of the refrigerator are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) The properties are to be obtained from the refrigerant tables (Tables A-11 through A-13):

\[
\begin{align*}
  h_1 &= h_g @ 200 \text{ kPa} = 244.46 \text{ kJ/kg} \\
  s_1 &= s_g @ 200 \text{ kPa} = 0.9377 \text{ kJ/kg.K} \\
  h_2 &= 263.30 \text{ kJ/kg} \\
  s_2 &= s_1 \\
  \eta_C &= \frac{h_2 - h_1}{h_2} \\
  \frac{0.80}{h_2 - 244.46} &= h_2 = 268.01 \text{ kJ/kg} \\
  h_3 &= h_f @ 500 \text{ kPa} = 73.33 \text{ kJ/kg} \\
  h_4 &= h_3 = 73.33 \text{ kJ/kg} \\
  h_5 &= h_g @ 400 \text{ kPa} = 255.55 \text{ kJ/kg} \\
  s_5 &= s_g @ 400 \text{ kPa} = 0.9269 \text{ kJ/kg.K} \\
  h_6 &= 278.33 \text{ kJ/kg} \\
  s_6 &= s_5 \\
  \eta_C &= \frac{h_6 - h_5}{h_6} \\
  \frac{0.80}{h_6 - 255.55} &= h_6 = 284.02 \text{ kJ/kg} \\
  h_7 &= h_f @ 1200 \text{ kPa} = 117.77 \text{ kJ/kg} \\
  h_8 &= h_7 = 117.77 \text{ kJ/kg} 
\end{align*}
\]

The mass flow rate of the refrigerant through the upper cycle is determined from an energy balance on the heat exchanger

\[
\dot{m}_A (h_5 - h_8) = \dot{m}_B (h_2 - h_3)
\]

\[
\dot{m}_A (255.55 - 117.77) \text{ kJ/kg} = (0.15 \text{ kg/s})(268.01 - 73.33) \text{ kJ/kg} \quad \rightarrow \quad \dot{m}_A = 0.212 \text{ kg/s}
\]

(b) The rate of heat removal from the refrigerated space is

\[
\dot{Q}_L = \dot{m}_B (h_1 - h_4) = (0.15 \text{ kg/s})(244.46 - 73.33) \text{ kJ/kg} = 25.67 \text{ kW}
\]

(c) The power input and the COP are

\[
\dot{W}_\text{in} = \dot{m}_A (h_6 - h_5) + \dot{m}_B (h_2 - h_1) \\
= (0.15 \text{ kg/s})(284.02 - 255.55) \text{ kJ/kg} + (0.212 \text{ kg/s})(268.01 - 244.46) \text{ kJ/kg} = 9.566 \text{ kW}
\]

\[
\text{COP} = \frac{\dot{Q}_L}{\dot{W}_\text{in}} = \frac{25.67}{9.566} = 2.68
\]
A two-stage cascade refrigeration cycle with a flash chamber is considered. The mass flow rate of the refrigerant through the high-pressure compressor, the rate of heat removal from the refrigerated space, the COP of the refrigerator, and the rate of heat removal and the COP if this refrigerator operated on a single-stage cycle between the same pressure limits are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Kinetic and potential energy changes are negligible.

**Analysis**

(a) The properties are to be obtained from the refrigerant tables (Tables A-11 through A-13):

\[
\begin{align*}
    h_1 &= h_{g@200 \text{ kPa}} = 244.46 \text{ kJ/kg} \\
    s_1 &= s_{g@200 \text{ kPa}} = 0.9377 \text{ kJ/kg.K} \\
    P_2 &= 450 \text{ kPa} \\
    s_2 &= s_1 \\
    \eta_C &= \frac{h_{2x} - h_1}{h_2 - h_1} \\
    &\approx \frac{261.09 - 244.46}{261.09 - 244.46} = 1.00 \\
    h_2 &= 265.24 \text{ kJ/kg} \\
\end{align*}
\]

\[
\begin{align*}
    h_3 &= h_{g@450 \text{ kPa}} = 257.53 \text{ kJ/kg} \\
    h_5 &= h_{f@1200 \text{ kPa}} = 117.177 \text{ kJ/kg} \\
    h_6 &= h_5 - 117.77 \text{ kJ/kg} \\
    h_7 &= h_{f@450 \text{ kPa}} = 68.81 \text{ kJ/kg} \\
    h_8 &= h_7 = 68.81 \text{ kJ/kg} \\
    h_9 &= h_6 = 117.77 \text{ kJ/kg} \\
    P_6 &= 450 \text{ kPa} \\
    \end{align*}
\]

The mass flow rate of the refrigerant through the high pressure compressor is determined from a mass balance on the flash chamber:

\[
\dot{m} = \frac{\dot{m}_7}{1-x_6} = \frac{0.15 \text{ kg/s}}{1-0.2594} = 0.2025 \text{ kg/s}
\]

Also, \[\dot{m}_3 = \dot{m} - \dot{m}_7 = 0.2025 - 0.15 = 0.05255 \text{ kg/s}\]

(b) The enthalpy at state 9 is determined from an energy balance on the mixing chamber:

\[
\begin{align*}
    \dot{m} h_9 &= \dot{m}_7 h_2 + \dot{m}_3 h_3 \\
    (0.2025 \text{ kg/s}) h_9 &= (0.15 \text{ kg/s})(265.24 \text{ kJ/kg}) + (0.05255 \text{ kg/s})(257.53 \text{ kJ/kg}) \\
    \Rightarrow h_9 &= 263.24 \text{ kJ/kg}
\end{align*}
\]

Then,

\[
\begin{align*}
    P_9 &= 450 \text{ kPa} \\
    h_9 &= 263.24 \text{ kJ/kg} \\
    s_9 &= 0.9451 \text{ kJ/kg.K} \\
    P_4 &= 1200 \text{ kPa} \\
    s_4 &= s_9 \\
    h_{4x} &= 284.27 \text{ kJ/kg} \\
    \eta_C &= \frac{h_{4x} - h_0}{h_4 - h_9} \\
    &\approx \frac{284.27 - 263.24}{289.53 - 263.24} = 1.00 \\
    h_4 &= 289.53 \text{ kJ/kg}
\end{align*}
\]
The rate of heat removal from the refrigerated space is
\[ \dot{Q}_L = \dot{m}_r (h_1 - h_8) = (0.15 \text{ kg/s})(244.46 - 68.81)\text{kJ/kg} = 26.35 \text{ kW} \]

(c) The power input and the COP are
\[
\dot{W}_{in} = \dot{m}_r (h_2 - h_1) + \dot{m}(h_4 - h_9) \\
= (0.15 \text{ kg/s})(265.24 - 244.46)\text{kJ/kg} + (0.2025 \text{ kg/s})(289.53 - 263.24)\text{kJ/kg} = 8.442 \text{ kW} \\
\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{26.35}{8.442} = 3.12
\]

(d) If this refrigerator operated on a single-stage cycle between the same pressure limits, we would have
\[
h_1 = h_{g@200 \text{ kPa}} = 244.46 \text{ kJ/kg} \\
s_1 = s_{g@200 \text{ kPa}} = 0.9377 \text{ kJ/kg.K} \\
P_2 = 1200 \text{ kPa} \\
s_2 = s_1 \rightarrow h_{2s} = 281.84 \text{ kJ/kg} \\
\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \\
0.80 = \frac{281.84 - 244.46}{h_2 - 244.46} \rightarrow h_2 = 291.19 \text{ kJ/kg} \\
h_3 = h_{f@1200 \text{ kPa}} = 117.77 \text{ kJ/kg} \\
h_4 = h_3 = 117.77 \text{ kJ/kg} \\
\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.2025 \text{ kg/s})(244.46 - 117.77)\text{kJ/kg} = 25.66 \text{ kW} \\
\dot{W}_{in} = \dot{m}(h_2 - h_1) = (0.2025 \text{ kg/s})(291.19 - 244.46)\text{kJ/kg} = 9.465 \text{ kW} \\
\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{25.66}{9.465} = 2.71